Black Hole Quasinormal Modes

Lecture 3

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References

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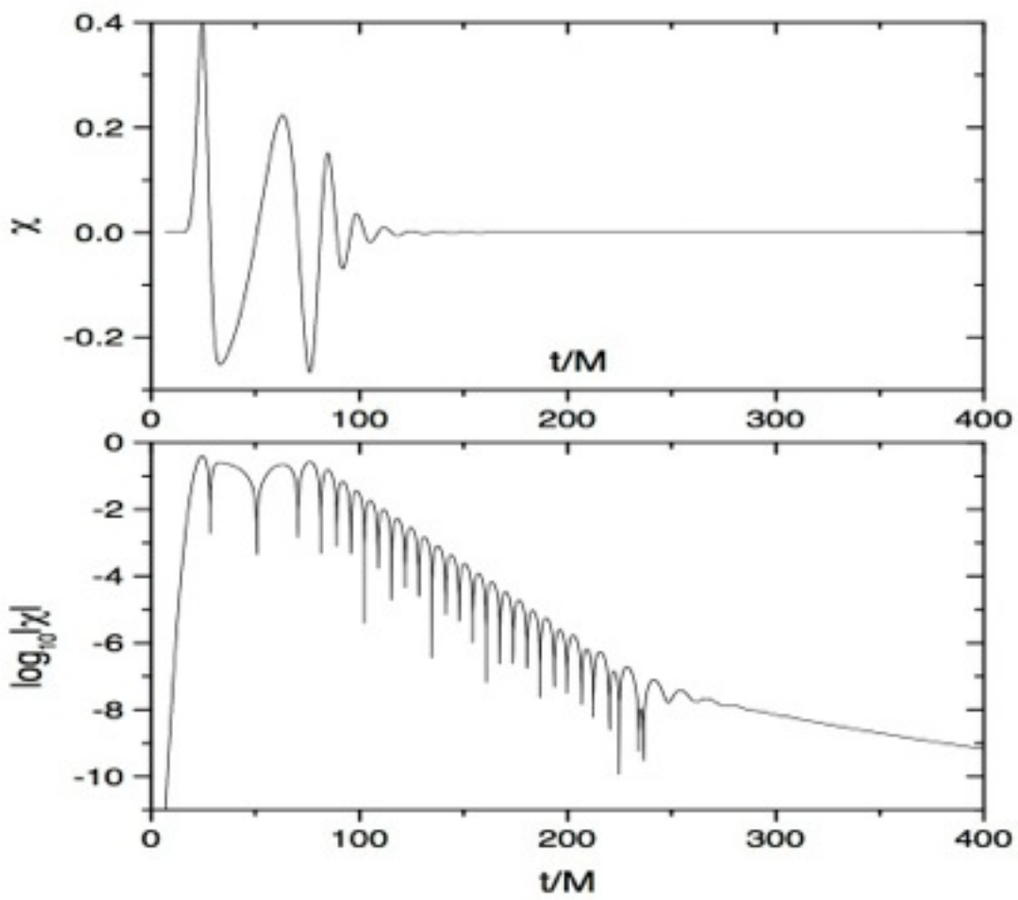
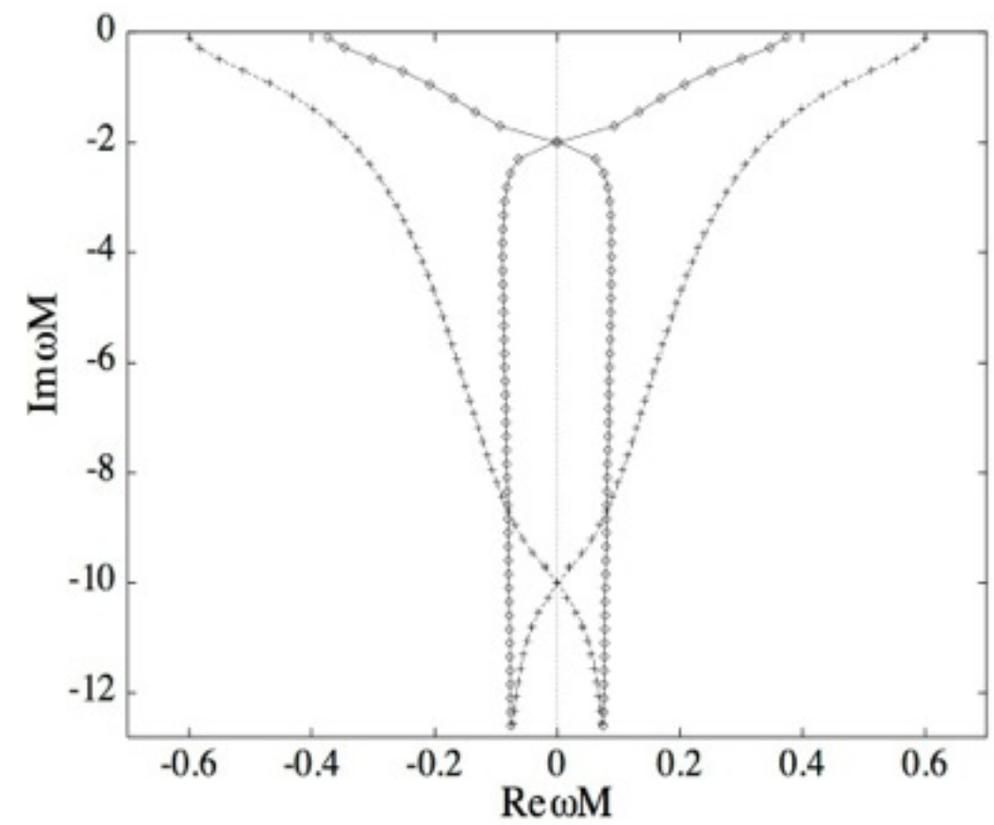
Schwarzschild BH





n	$\ell = 2$		$\ell = 3$		$\ell = 4$	
0	0.37367	-0.08896 i	0.59944	-0.09270 i	0.80918	-0.09416 i
1	0.34671	-0.27391 i	0.58264	-0.28130 i	0.79663	-0.28443 i
2	0.30105	-0.47828 i	0.55168	-0.47909 i	0.77271	-0.47991 i
3	0.25150	-0.70514 i	0.51196	-0.69034 i	0.73984	-0.68392 i

$$2\pi(5142\,\mathrm{Hz}) \times (M_\odot/M)$$

$$f = 12.07\,\mathrm{kHz}$$

$$\tau = 0.35\,\mathrm{ms}$$

Kerr Black Hole

- Kerr metric

$$
\begin{aligned}
ds^2 &= -\left(1-\frac{2Mr}{\Sigma}\right)dt^2 - \frac{4Mar\sin^2\theta}{\Sigma}dtd\varphi + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \\
&\quad + \left(r^2 + a^2 + \frac{2Ma^2r\sin^2\theta}{\Sigma}\right)\sin^2\theta d\varphi^2,
\end{aligned}
$$

with

$$
\Delta = r^2 - 2Mr + a^2, \qquad \Sigma = r^2 + a^2\cos^2\theta.
$$

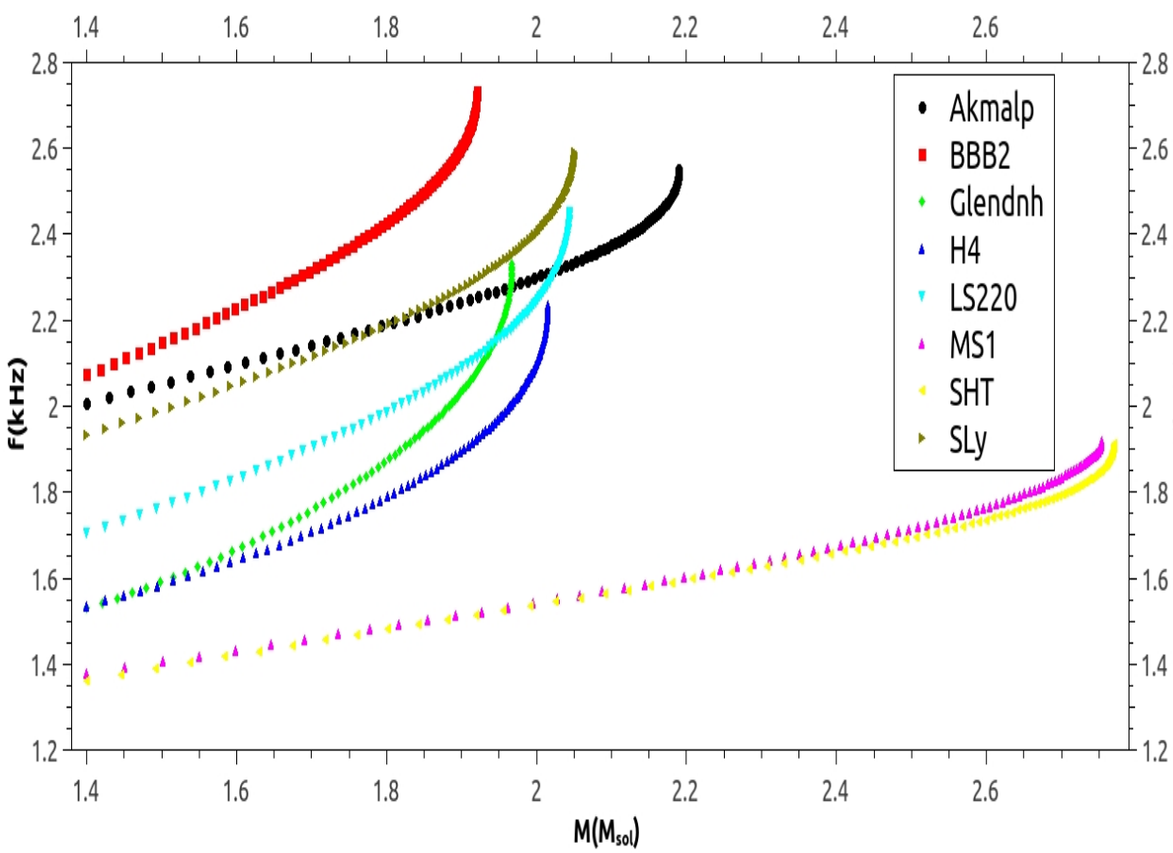
- Teukolsky equation (1972)

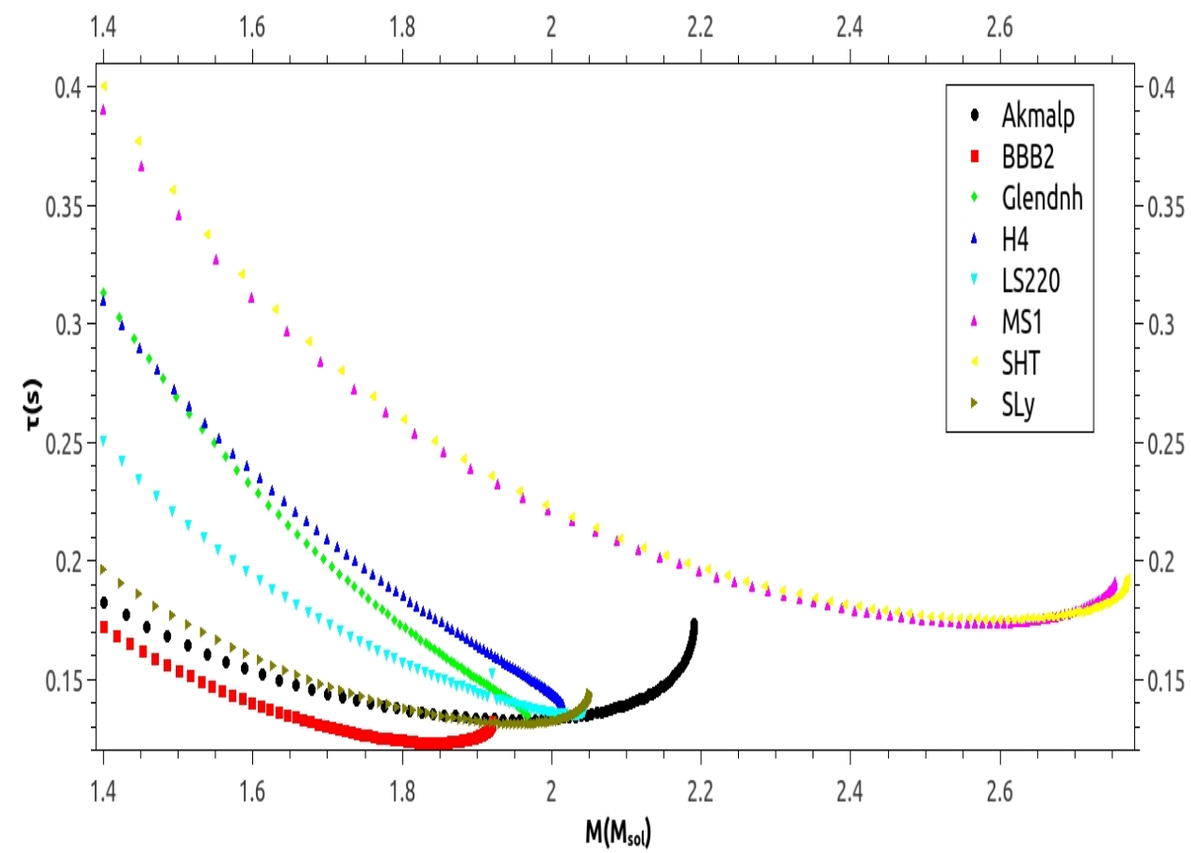
$$
\begin{aligned}
&\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\chi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\chi}{\partial t\partial\varphi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\chi}{\partial\varphi^2} \\
&-\Delta^{-\sigma}\frac{\partial}{\partial r}\left(\Delta^{\sigma+1}\frac{\partial\chi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\chi}{\partial\theta}\right) - 2\sigma\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\chi}{\partial\varphi} \\
&-2\sigma\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\chi}{\partial t} + \left(\sigma^2\cot^2\theta - \sigma\right)\chi = 0
\end{aligned}
$$

- Using the Pöschl-Teller formula: $\omega \approx \frac{1}{3\sqrt{3}M}\left(\pm\left(\ell+\frac{1}{2}\right) + \frac{2am}{3\sqrt{3}M} - \left(n+\frac{1}{2}\right)i\right)$

Neutron Stars

- Frequencies and damping times for the fundamental mode (EOS-dependent!)





What can we do with QNMs?

- Linear stability analysis
- Study the frequency spectrum
- AdS/CFT correspondence
- If detected, QNMs can be used to distinguish between different sources…

Gravitational waves detected!

PRL **116,** 061102 (2016)

Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW LETTERS

week ending
12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4}M_\odot$ and $29^{+4}_{-4}M_\odot$, and the final black hole mass is $62^{+4}_{-4}M_\odot$, with $3.0^{+0.5}_{-0.5}M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

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I. INTRODUCTION

In 1916, the year after the final formulation of the field equations of general relativity, Albert Einstein predicted the existence of gravitational waves. He found that the linearized weak-field equations had wave solutions: transverse waves of spatial strain that travel at the speed of light, generated by time variations of the mass quadrupole moment of the source [1,2]. Einstein understood that gravitational-wave amplitudes would be remarkably small; moreover, until the Chapel Hill conference in 1957 there was significant debate about the physical reality of gravitational waves [3].

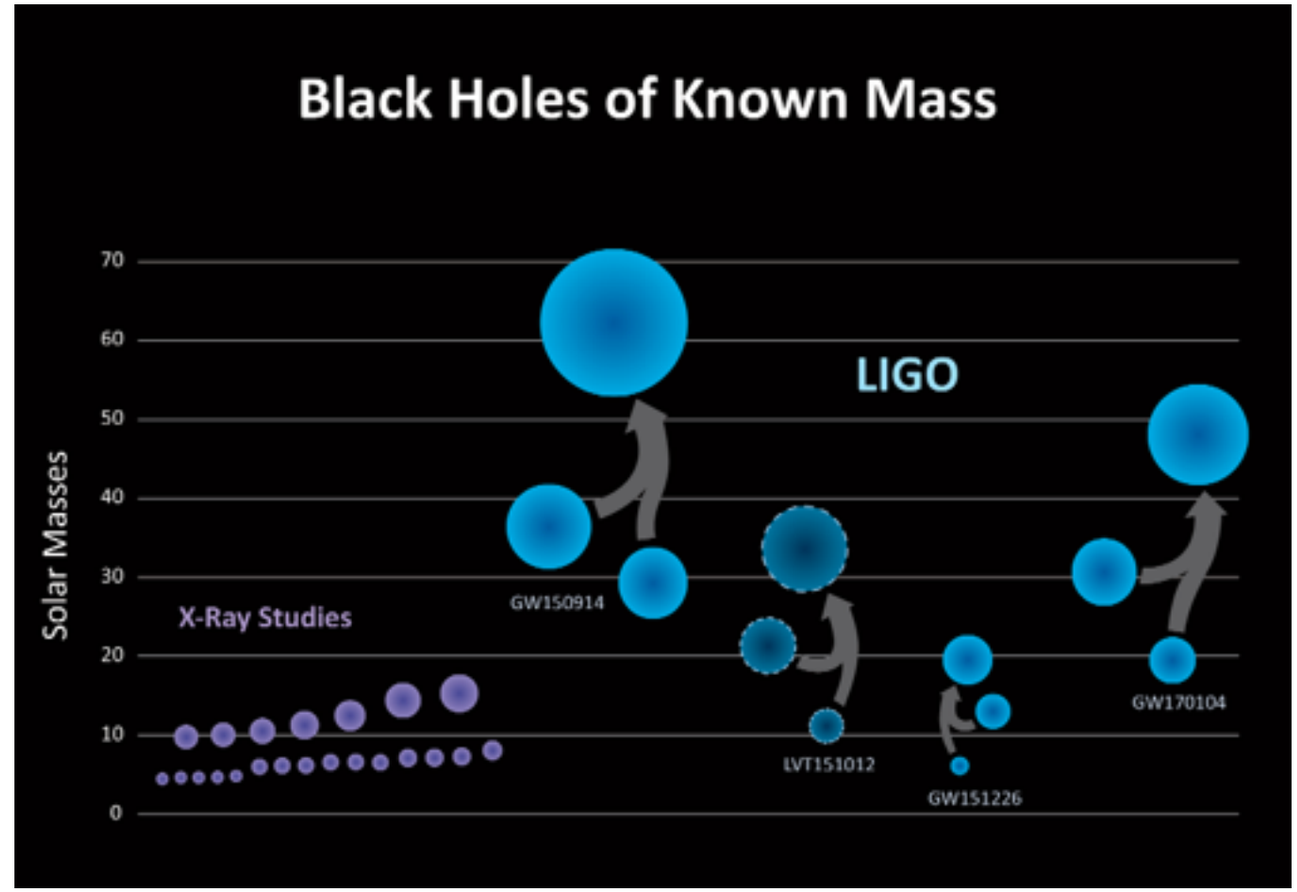
Also in 1916, Schwarzschild published a solution for the field equations [4] that was later understood to describe a black hole [5,6], and in 1963 Kerr generalized the solution to rotating black holes [7]. Starting in the 1970s theoretical work led to the understanding of black hole quasinormal modes [8–10], and in the 1990s higher-order post-Newtonian calculations [11] preceded extensive analytical studies of relativistic two-body dynamics [12,13]. These advances, together with numerical relativity breakthroughs in the past decade [14–16], have enabled modeling of binary black hole mergers and accurate predictions of their gravitational waveforms. While numerous black hole candidates have now been identified through electromagnetic observations [17–19], black hole mergers have not previously been observed.

The discovery of the binary pulsar system PSR B1913+16 by Hulse and Taylor [20] and subsequent observations of its energy loss by Taylor and Weisberg [21] demonstrated the existence of gravitational waves. This discovery, along with emerging astrophysical understanding [22], led to the recognition that direct observations of the amplitude and phase of gravitational waves would enable studies of additional relativistic systems and provide new tests of general relativity, especially in the dynamic strong-field regime.

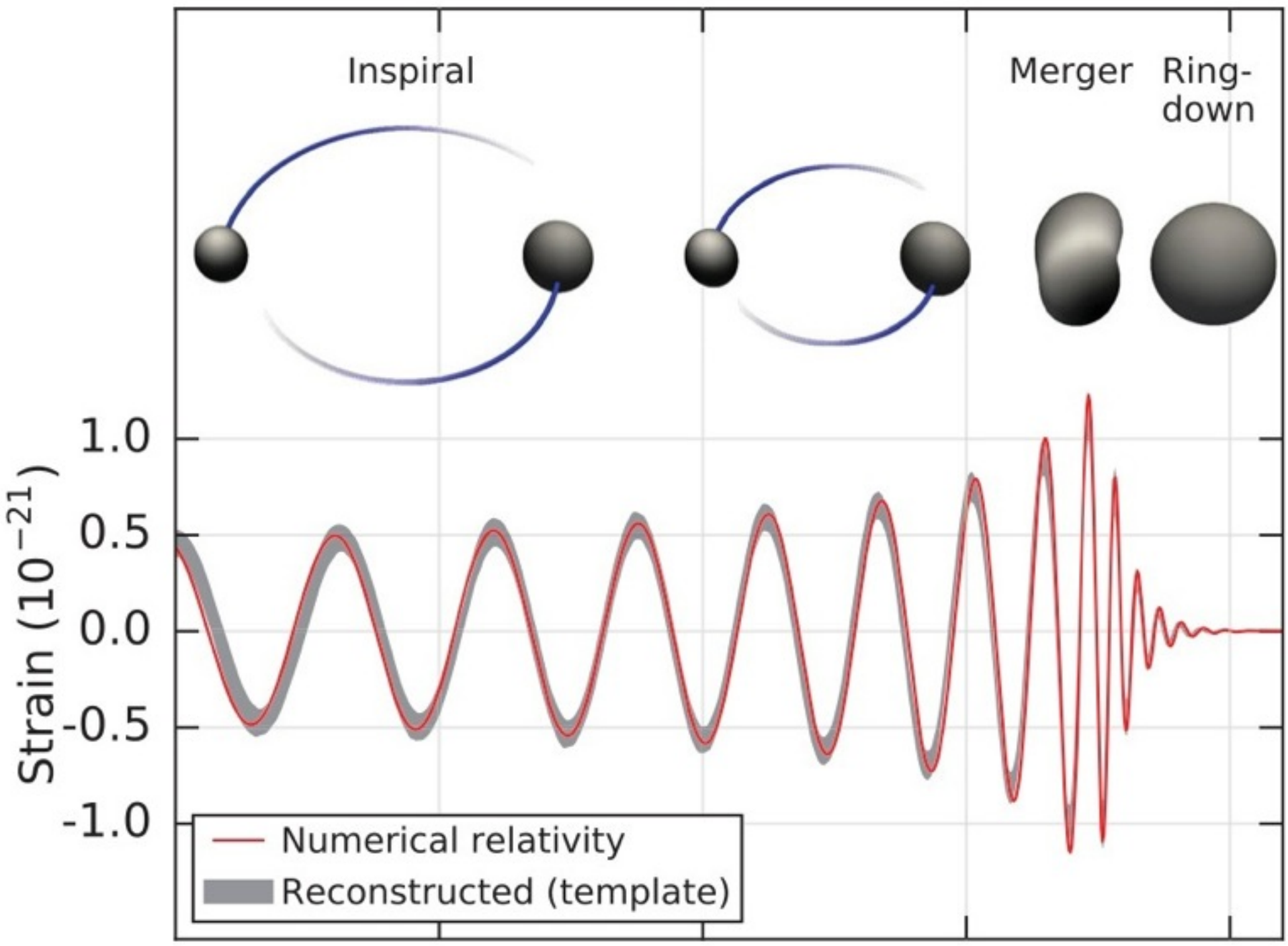
Experiments to detect gravitational waves began with Weber and his resonant mass detectors in the 1960s [23], followed by an international network of cryogenic resonant detectors [24]. Interferometric detectors were first suggested in the early 1960s [25] and the 1970s [26]. A study of the noise and performance of such detectors [27], and further concepts to improve them [28], led to proposals for long-baseline broadband laser interferometers with the potential for significantly increased sensitivity [29–32]. By the early 2000s, a set of initial detectors was completed, including TAMA 300 in Japan, GEO 600 in Germany, the Laser Interferometer Gravitational-Wave Observatory (LIGO) in the United States, and Virgo in Italy. Combinations of these detectors made joint observations from 2002 through 2011, setting upper limits on a variety of gravitational-wave sources while evolving into a global network. In 2015, Advanced LIGO became the first of a significantly more sensitive network of advanced detectors to begin observations [33–36].

A century after the fundamental predictions of Einstein and Schwarzschild, we report the first direct detection of gravitational waves and the first direct observation of a binary black hole system merging to form a single black hole. Our observations provide unique access to the

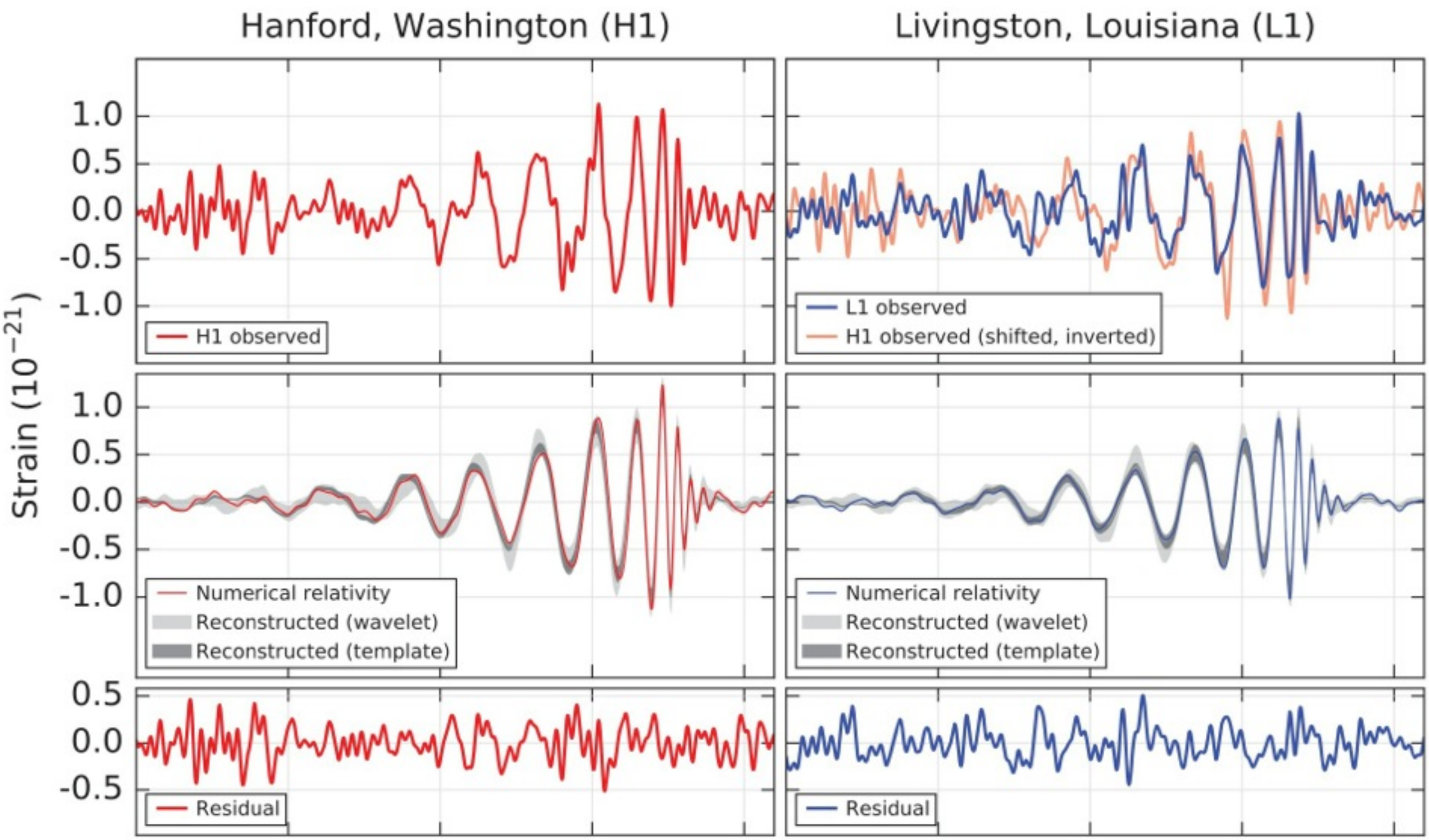
[*]Full author list given at the end of the article.



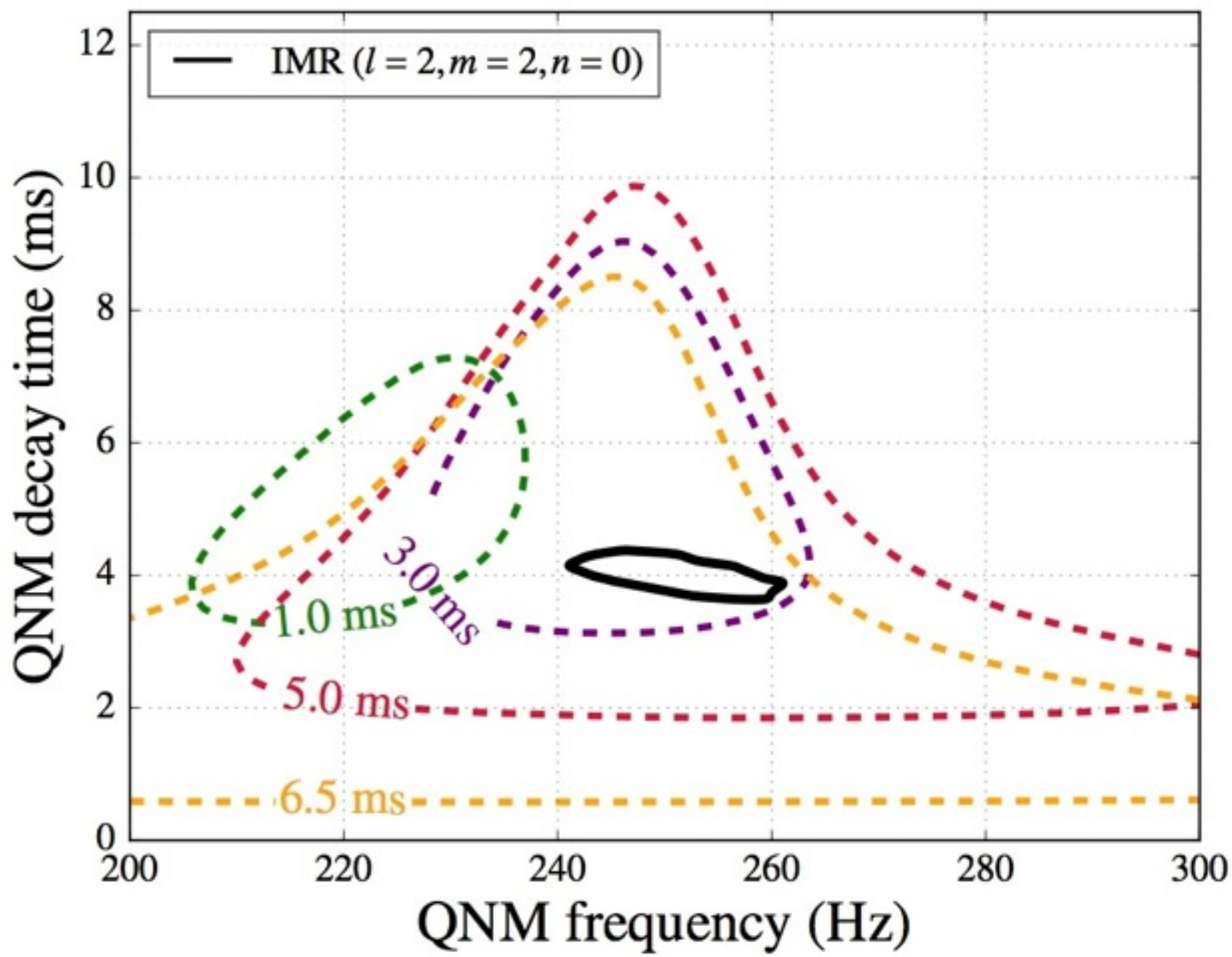
Binary black holes



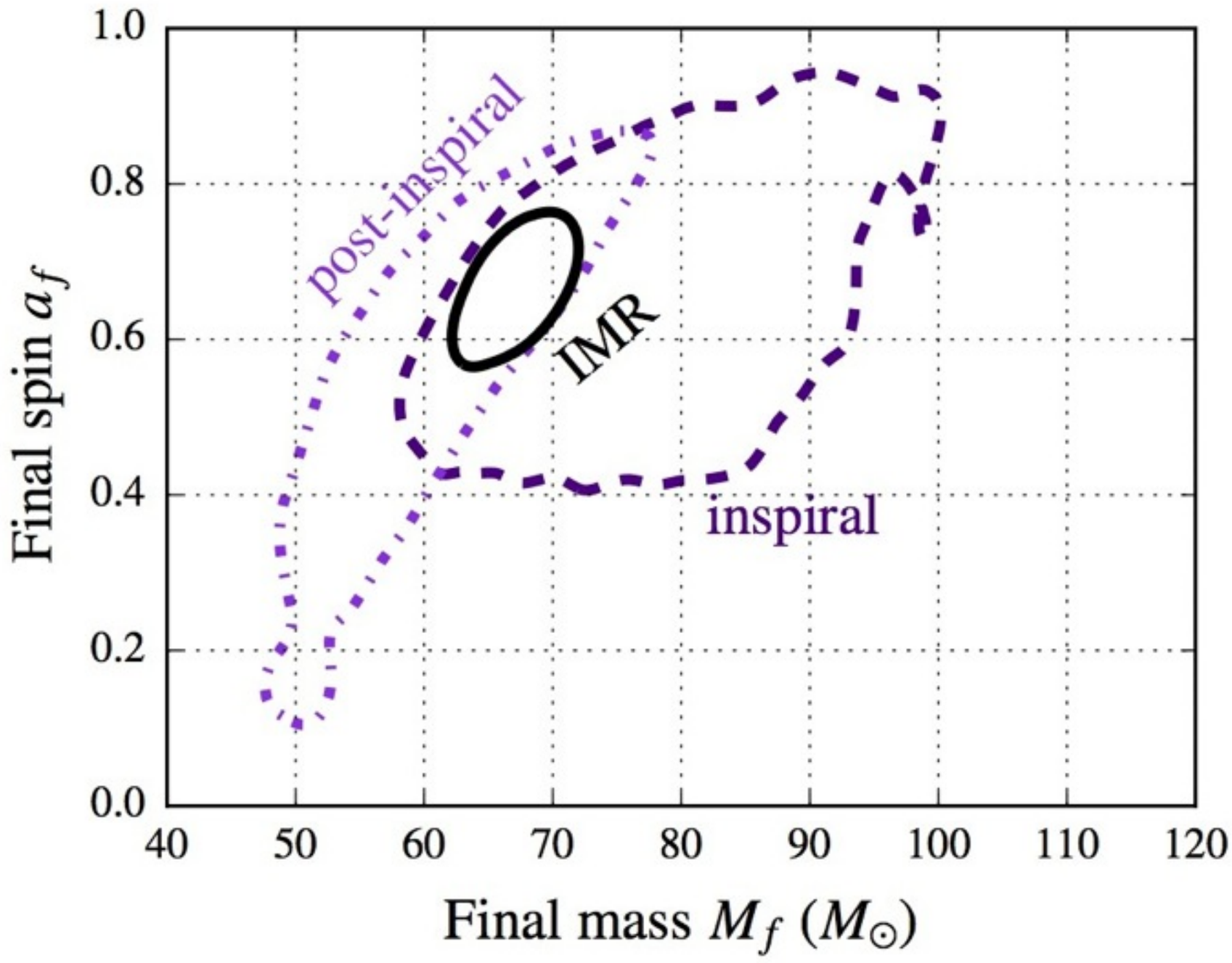
GW150914


Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual

Lowest damped QNM



Final Black Hole



Tests of GR with QNMs

- Needs detection of the fundamental mode and first overtone
- Independent determination of mass and spin from both modes
- Allows a test of the no-hair theorem!

Black Hole alternatives

- why do we need BH alternatives?
- what is a BH alternative?

Black Hole alternatives

- What is a BH alternative?
- Why do we need BH alternatives? What should we do with them?
- possibilities:
  - wormholes
  - boson stars
  - gravastars
  - superspinars, etc

Example of a black hole alternative: the gravastar model

- Alternative to the end state of stellar evolution
- "gravitational vacuum condensate star"
- almost as compact as a black hole
- no central singularity or event horizon
- Formation in the collapse of a massive star

$$\mu = M/r_2$$

$$\delta = r_2 - r_1$$

Test: could GW150914 have created a gravastar?