

Introduction

The universal gravitation describes the movement of bodies under gravity's action. To solve the n-body problem of it's necessary to find an expression for the position to each body at a given time. In order to do this we use Newton's law of Universal Gravitation

$$\vec{F} = - \sum_{j=1}^n Gm_i m_j \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|^3} \quad j \neq i. \quad (1)$$

The Java software Triana [1], developed by the Cardiff University, and the software Gnuplot [2] were also used to plot for the solutions. The usage of Triana is interesting because it uses an approximation of the movement of the bodies in each time-step as Uniformly Accelerated Movement, with accuracy determined by the user.

The present work starts with the central force problem, Kepler's laws, and then studies the two and three body problems, as preliminary steps to the n-body problem, and uses as main reference the book Gravity from the ground up [3].

Result and discussions

The trajectory of the bodys are showed in each case.

Central Force Problem

The central force problem can be solved through the Orbit program from the Triana software. This program allows the user to solve Newton's laws for the motion of planets in the gravitational field of a central mass, by making the gravitational force (equation 1) that depends on the radial distance r as $1/r^2$.

The user must set the time-step Δt , the initial velocity v_0 , the inicial position x_0 and y_0 , and the mass m_j of the central body, as well as the relative error he can tolerate in assuming a constant acceleration. With the use of the initial x-component of the radial distance x_0 on equation 2 is defined as the velocity change $\Delta v = a_0 \Delta t$, which defines the position change $\Delta x = ((v_0 + v_0 + \Delta v)/2) \Delta t$, and once pthe desired accuracy is met, the new position is $x = x_0 + \Delta x$. All of this is done in the same way for y-component as well as x. This calculation is repeated as many times as the user defines or until the orbit closes.

$$\vec{a} = - \frac{Gm_j}{r^3} \vec{r}. \quad (2)$$

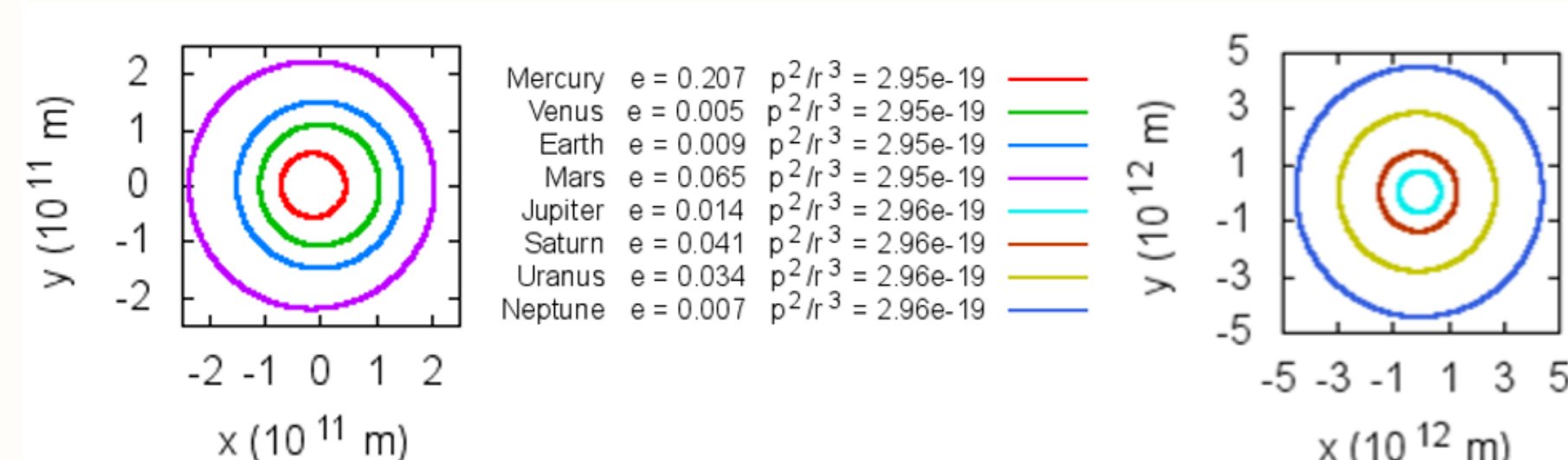


Figure 1: Orbits of the solar system with the eccentricity and constant p^2/r^3 for each planet

Depending on the velocity input by the user the orbit can be closed or not, as shown in figure 2. To discover the escape velocity we use equation 3.

$$\vec{v}_i = \sqrt{\frac{2GM}{r_i}} \quad (3)$$

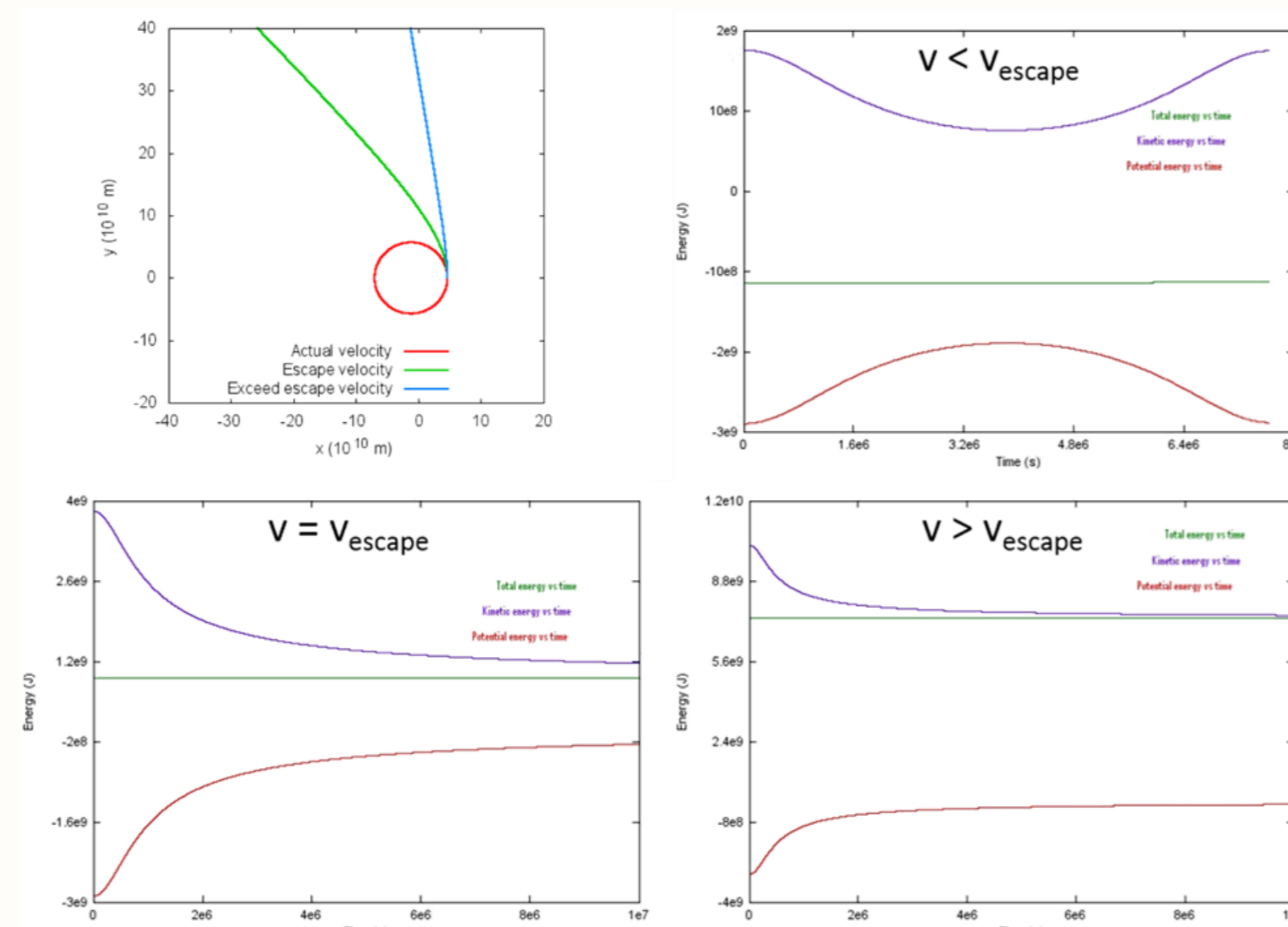


Figure 2: Orbits of Mercury with its actual (red), escape (green) and exceed escape velocity (blue) into the Sun's gravitational field, and their energy plots.

It's also possible to verify the Kepler's laws with this code, and to see the energy conservation of the system. The first law states that the planet's orbit should be elliptical, and the eccentricity of the closed orbits of each planet are showed in figure 1. The second law states that the area swept per unit time is a constant, which is calculated at each time-step of the program and the results appear in figure 3. And the third law states that the square of the period divided by the cube of the orbit average ratio is also a constant, which is calculated at the end of the program and is also shown in figure 1.

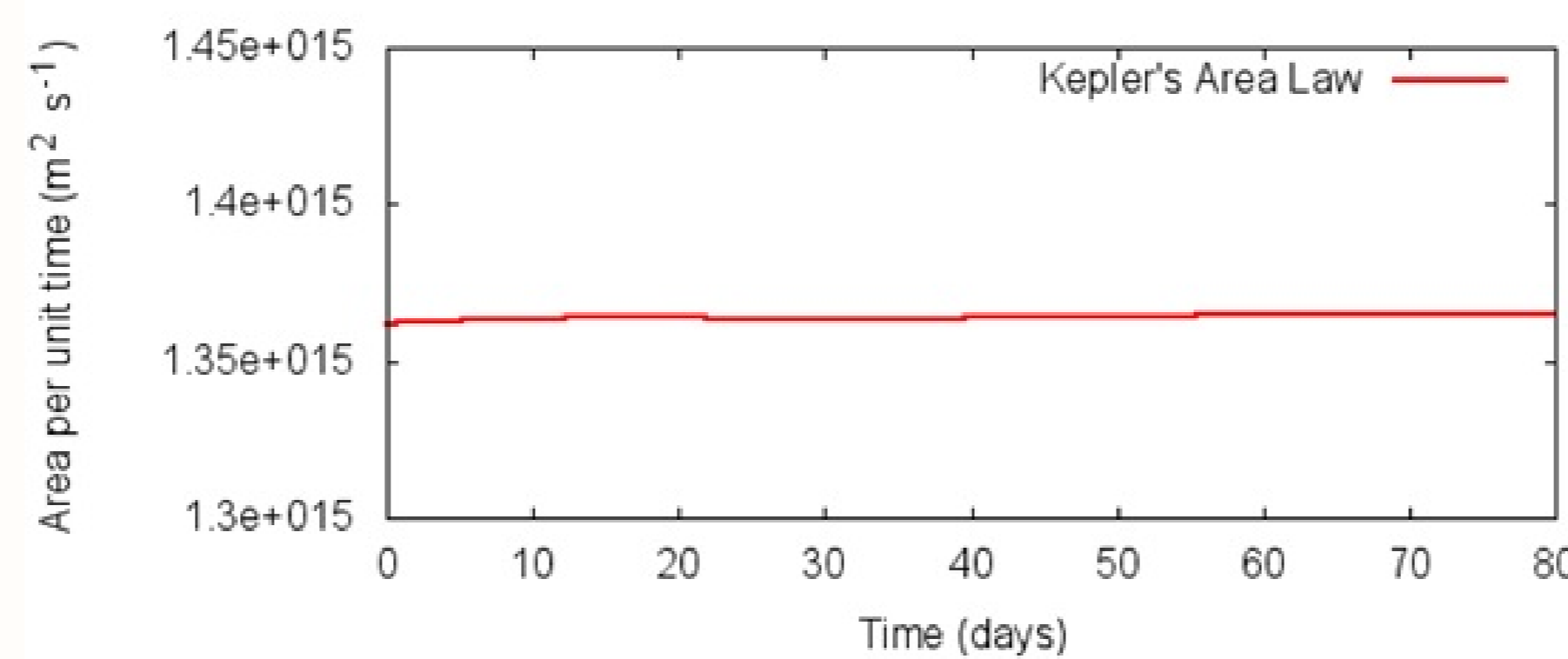


Figure 3: Numerical verification of Kepler's Area Law (the second law) for Mercury.

Two body problem

The two body problem can be solved through the Binary program from the Triana software. The code used also contains a loop, where it works in the same way as Orbit, except that the central body is no longer at rest. So it's necessary to do the calculations for both bodies, to obtain both positions after each time-step.

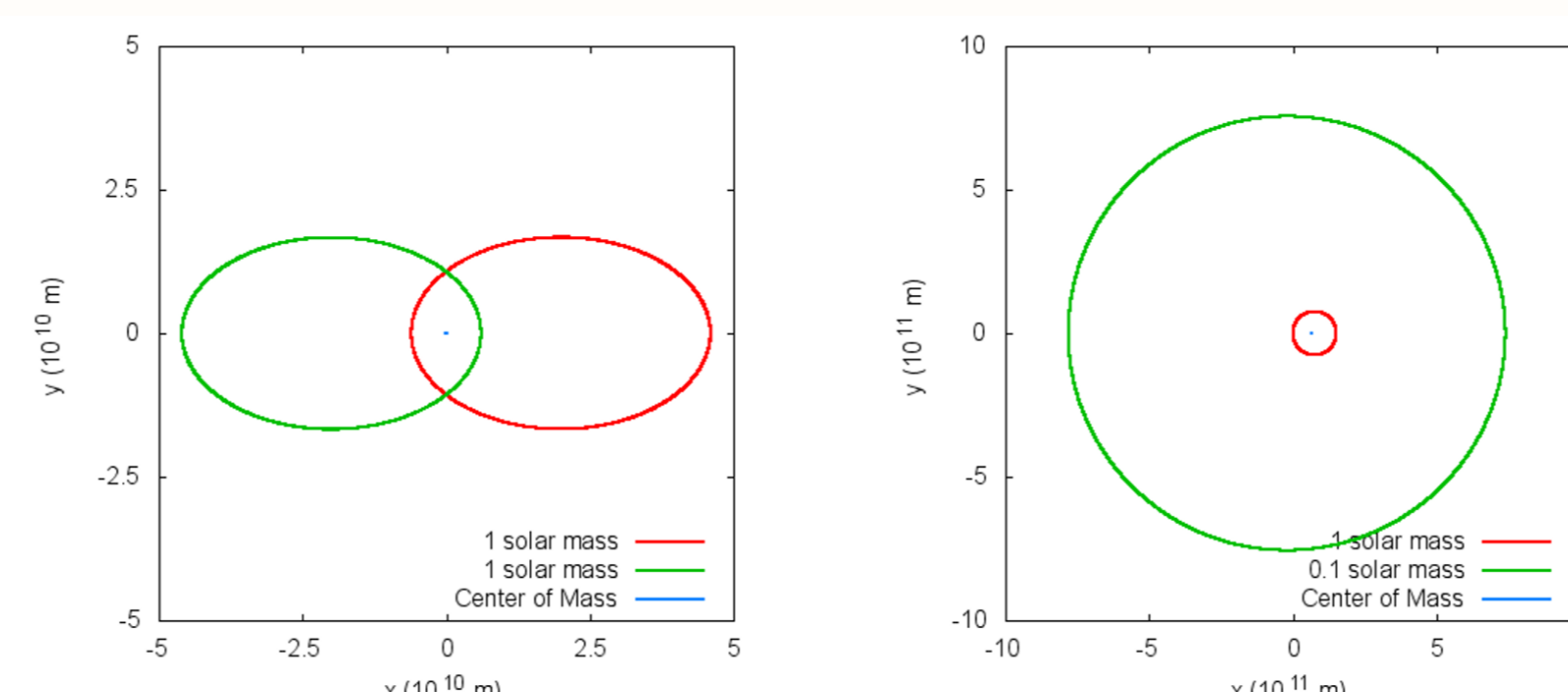


Figure 4: Orbits of a closed binary system with zero total momentum.

It's also possible to verify that in a closed system the momentum of the center of mass is conserved, because the external force is zero in this case:

$$F_{\text{external}} = M \frac{d^2 R}{dt^2} = \frac{dP}{dt} = 0. \quad (4)$$

So the center of mass can be at rest when the total momentum of the system is zero (figure 4), or it can move

with constant velocity, when the total momentum of the system is non-zero (figure 5).

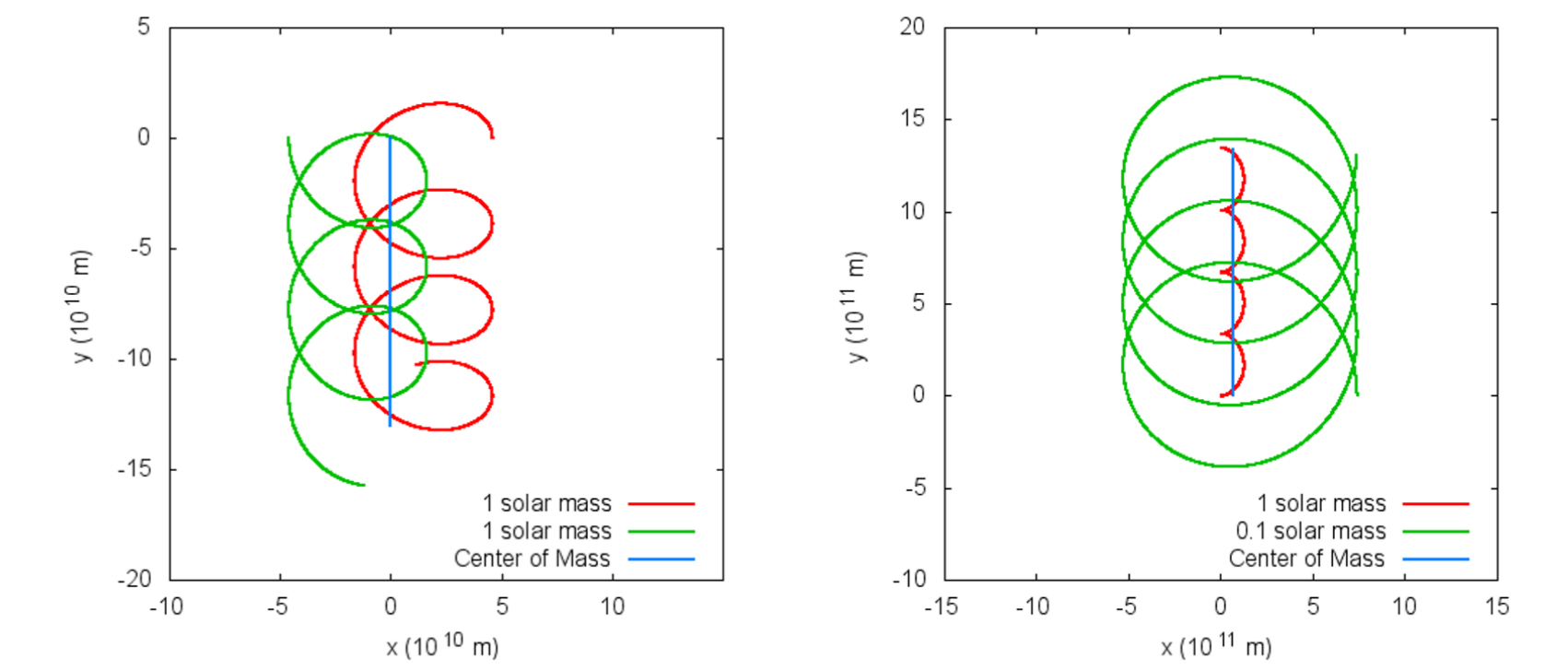


Figure 5: Orbits of a closed binary system with non-zero total momentum.

Three body problem

The three body problem can be solved through the MercPert and Multiple programs from the Triana software. The code of MercPert uses a loop that calculates the movement of a particle with a negligible mass caused by the gravitational force of a binary system. The code used in Multiple contains two loops, where the same calculations made in Binary are performed, but for each pair of bodies in the system. Multiple can also be used in the n-body problem.

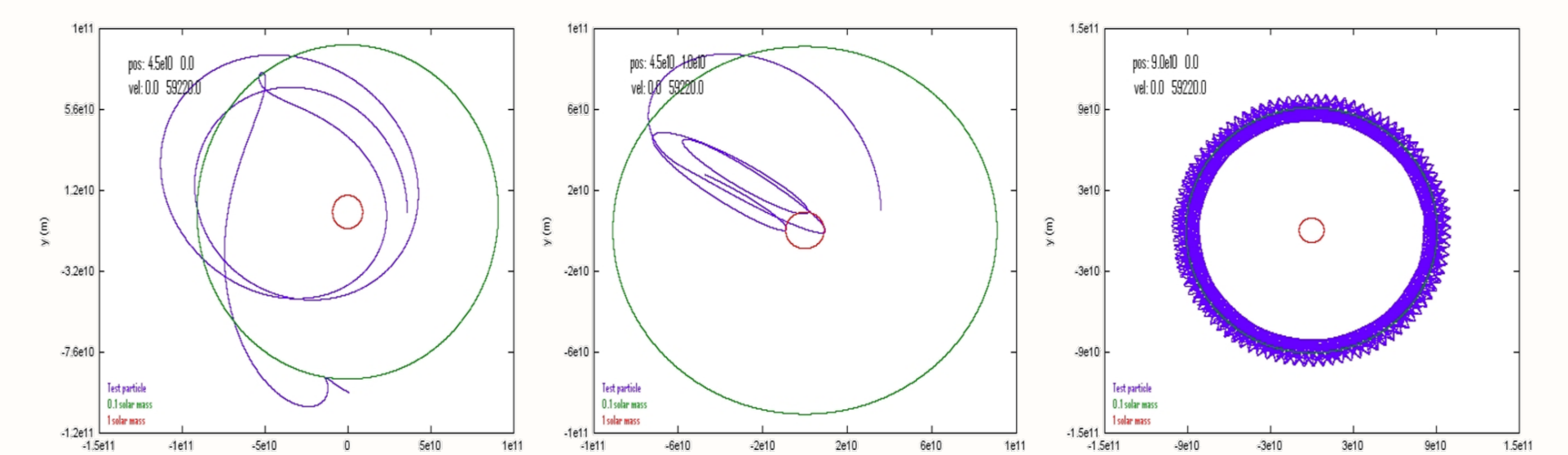


Figure 6: Orbits of the restricted three body problem. Note that the orbits aren't closed.

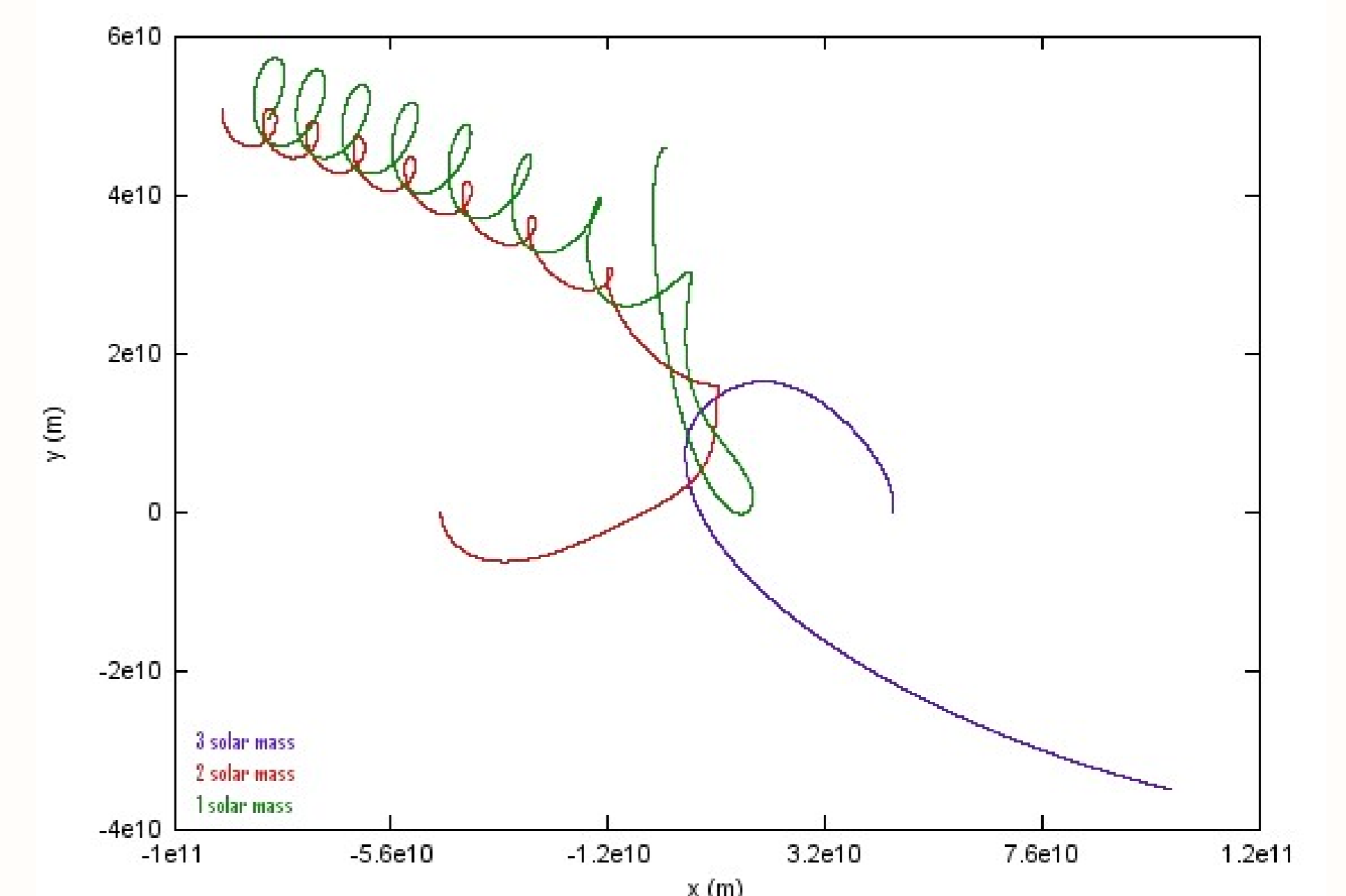


Figure 7: Result of a three body interaction where a binary system is disrupted by the third body.

Summary and conclusions

With the use of basic physics it is possible to plot numerical solutions of physical problems that require an advanced knowledge in mechanics. Using this same software we expect to achieve numerical results for the n-body problem.

References

- [1] Triana: The open source environment. <http://www.trianacode.org/>. Accessed: 2015-05-12.
- [2] Gnuplot homepage. <http://www.gnuplot.info/>. Accessed: 2015-05-12.
- [3] Bernard Schutz. *Gravity from the Ground Up*. Press Syndicate of the University of Cambridge, 2005.