UFABC

FOLHA DE PROVA

NOTA

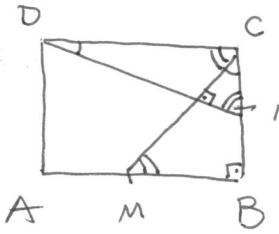
Nome: _____

R.A. _____ Data: ____ / ____ / ____

Disciplina: _____ Cód. Disciplina: _____

Professor: _____

(2)



$\overrightarrow{AB} = \vec{a}$

$\overrightarrow{AD} = \vec{b}$

Δ_{CDN} e Δ_{BCM} : congruentes

Δ_{CIN} e Δ_{BCM} : semelhantes

$$\overrightarrow{DN} = \overrightarrow{DC} + \overrightarrow{CN} = \vec{a} - \frac{\vec{b}}{2}$$

$$\overrightarrow{MC} = \overrightarrow{MB} + \overrightarrow{BC} = \frac{\vec{a}}{2} + \vec{b}$$

$$|\overrightarrow{DN}| = |\overrightarrow{MC}| = \sqrt{\left(\frac{\vec{a}}{2} + \vec{b}\right) \cdot \left(\frac{\vec{a}}{2} + \vec{b}\right)}$$

$$= \sqrt{\frac{|\vec{a}|^2}{4} + 2\,\frac{\vec{a} \cdot \vec{b}}{2} + |\vec{b}|^2} = \sqrt{\frac{|\vec{a}|^2}{4} + |\vec{a}|^2}$$

($\vec{a} \cdot \vec{b} = 0$)

$$= \sqrt{\frac{5|\vec{a}|^2}{4}} = \frac{\sqrt{5}}{2}|\vec{a}|$$

$$\overrightarrow{IN} = \alpha \overrightarrow{DN} \Rightarrow \overrightarrow{IN} = \alpha\left(\vec{a} - \frac{\vec{b}}{2}\right) \text{ e } |\overrightarrow{IN}| = \frac{\alpha\sqrt{5}}{2}|\vec{a}|$$

$$\overrightarrow{IC} = \beta \overrightarrow{MC} \Rightarrow \overrightarrow{IC} = \beta\left(\frac{\vec{a}}{2} + \vec{b}\right) \text{ e } |\overrightarrow{IC}| = \frac{\beta\sqrt{5}}{2}|\vec{a}|$$

$$\frac{|\overrightarrow{IC}|}{|\overrightarrow{BC}|} = \frac{|\overrightarrow{IN}|}{|\overrightarrow{BM}|} = \frac{|\overrightarrow{CN}|}{|\overrightarrow{CM}|}$$

$$\frac{|\overrightarrow{CN}|}{|\overrightarrow{CM}|} = \frac{\frac{|\overrightarrow{b}|}{2}}{\frac{\sqrt{5}\,|\overrightarrow{a}|}{2}} = \frac{1}{\sqrt{5}}$$

$$\frac{|\overrightarrow{IC}|}{|\overrightarrow{BC}|} = \frac{1}{\sqrt{5}} \Rightarrow \frac{\beta\sqrt{5}}{2}\,\frac{|\overrightarrow{a}|}{|\overrightarrow{b}|} = \frac{1}{\sqrt{5}} \Rightarrow \boxed{\beta = \frac{2}{5}}$$

$$\frac{|\overrightarrow{IN}|}{|\overrightarrow{BM}|} = \frac{1}{\sqrt{5}} \Rightarrow \frac{\alpha\sqrt{5}}{2}\,\frac{|\overrightarrow{a}|}{\frac{|\overrightarrow{a}|}{2}} = \frac{1}{\sqrt{5}} \Rightarrow \boxed{\alpha = \frac{1}{5}}$$

$$\overrightarrow{IC} = \frac{2}{5}\left(\frac{\overrightarrow{a}}{2} + \overrightarrow{b}\right) \Rightarrow \boxed{\overrightarrow{IC} = \frac{1}{5}\left(\overrightarrow{a} + 2\overrightarrow{b}\right)} \quad (a)$$

$$\overrightarrow{IN} = \frac{1}{5}\left(\overrightarrow{a} - \frac{\overrightarrow{b}}{2}\right)$$

$$\text{área } \Delta CIN = \frac{|\overrightarrow{IN}| \cdot |\overrightarrow{IC}|}{2} = \frac{1}{5}\,\frac{\sqrt{5}}{2}\,|\overrightarrow{a}| \cdot \frac{2}{5}\,\frac{\sqrt{5}}{2}\,|\overrightarrow{a}|\,\frac{1}{2}$$

$$\therefore \boxed{\text{área } \Delta CIN = \frac{|\overrightarrow{a}|^2}{20}} \quad (b)$$

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(3) a) $[\vec{u}+\vec{v}, \vec{v}+\vec{w}, \vec{u}+\vec{w}] = 2[\vec{u}, \vec{v}, \vec{w}]$

$[\vec{u}+\vec{v}, \vec{v}+\vec{w}, \vec{u}+\vec{w}] =$

$= [\vec{u}, \vec{v}+\vec{w}, \vec{u}+\vec{w}] + [\vec{v}, \vec{v}+\vec{w}, \vec{u}+\vec{w}]$

$= [\vec{u}, \vec{v}, \vec{u}+\vec{w}] + [\vec{u}, \vec{w}, \vec{u}+\vec{w}] +$ (0)

$+ [\vec{v}, \vec{v}, \vec{u}+\vec{w}] + [\vec{v}, \vec{w}, \vec{u}+\vec{w}]$ (0)

$= [\vec{u}, \vec{v}, \vec{u}] + [\vec{u}, \vec{v}, \vec{w}] +$ (0)

$+ [\vec{v}, \vec{w}, \vec{u}] + [\vec{v}, \vec{w}, \vec{w}]$ (0)

$= [\vec{u}, \vec{v}, \vec{w}] + [\vec{v}, \vec{w}, \vec{u}] = 2[\vec{u}, \vec{v}, \vec{w}]$

ABSQUE SUDORE ET LABORE NULLUM OPUS PERFECTUM EST
"SEM SUOR E SEM TRABALHO NENHUMA OBRA É TERMINADA"
SCHREVELIUS 1176

b) $[\vec{a}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$

$[\vec{a}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] =$

$= [\vec{a}, \vec{a}, \vec{a}+\vec{b}+\vec{c}] + [\vec{a}, \vec{b}, \vec{a}+\vec{b}+\vec{c}]$

$= [\vec{a}, \vec{b}, \vec{a}] + [\vec{a}, \vec{b}, \vec{b}+\vec{c}]$

$= [\vec{a}, \vec{b}, \vec{b}] + [\vec{a}, \vec{b}, \vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$ ✗

c) $|[\vec{u}, \vec{v}, \vec{w}]| = |\vec{u} \times \vec{v} \cdot \vec{w}|$

$= ||\vec{u} \times \vec{v}| \cdot |\vec{w}| \cos\alpha|$

$= ||\vec{u}| \cdot |\vec{v}| \operatorname{sen}\beta |\vec{w}| \cos\alpha|$

$= |\vec{u}| \cdot |\vec{v}| \cdot |\vec{w}| \cdot |\operatorname{sen}\beta \cos\alpha|$

$= |\vec{u}| \cdot |\vec{v}| \cdot |\vec{w}|$, se $\alpha = 0$ ou π

e $\beta = \frac{\pi}{2}$ ou $-\frac{\pi}{2}$

$\alpha = 0 \Rightarrow \vec{w} \parallel \vec{u} \times \vec{v}$

$\beta = \frac{\pi}{2} \Rightarrow \vec{u} \perp \vec{v}$