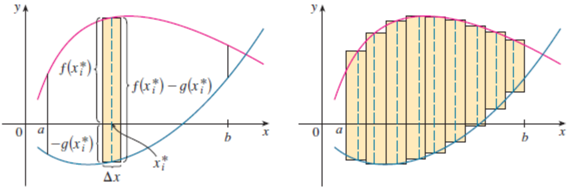
Integrais - Aplicações I

Daniel

2 de dezembro de 2014

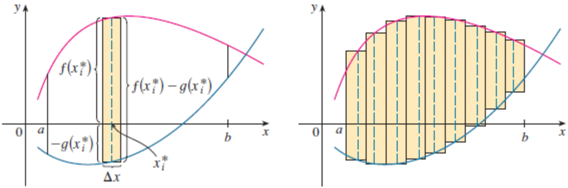
Área entre duas curvas.

Figura: 

$$A = \lim_{n \to \infty} \sum_{i=1}^{\infty} (f(x_i^*) - g(x_i^*)) \Delta x_i \tag{1}$$

$$A = \int_a^b (f(x) - g(x))\, dx \tag{2}$$

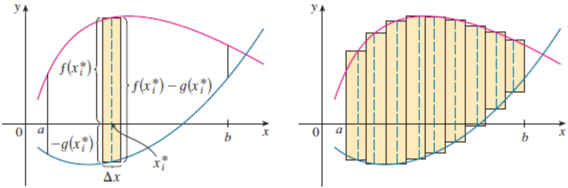
Área entre duas curvas.

Figura: 

$$A = \lim_{n\to\infty} \sum_{i=1}^{\infty} \left(f(x_i^*) - g(x_i^*) \right) \Delta x_i \qquad (1)$$

$$A = \int_a^b \left(f(x) - g(x) \right) dx \qquad (2)$$

Área entre duas curvas.

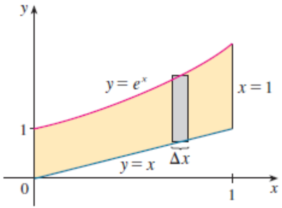
Figura: 

$$A = \lim_{n\to\infty} \sum_{i=1}^{\infty} \left(f(x_i^*) - g(x_i^*)\right) \Delta x_i \qquad (1)$$

$$A = \int_a^b \left(f(x) - g(x)\right) dx \qquad (2)$$

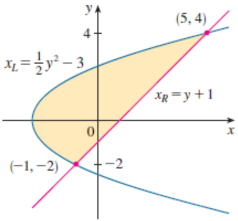
Exemplo 1

Ache a área do gráfico delimitado por $y = e^x$, $y = x$, $x = 0$ e $x = 1$

Figura: 

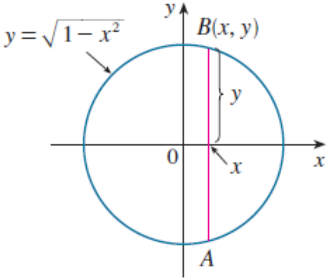
Exemplo 2

Ache a área delimitada pelas curvas $y = x - 1$ e a parábola $y^2 = 2x + 6$

Figura: 

Exemplo 3 - Área do Círculo

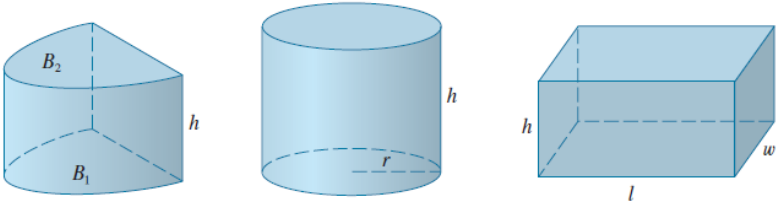
Aréa do Círculo



Volume por Seções Transversais

Volume de Cilindros Retos: Aréa da Base x Altura

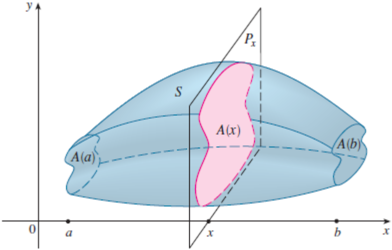
Figura:

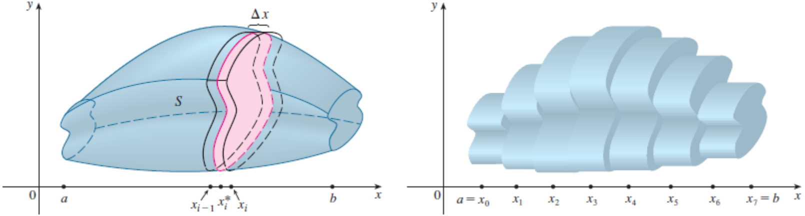


(a) Cylinder
$V = Ah$

(b) Circular cylinder
$V = \pi r^2 h$

(c) Rectangular box
$V = lwh$

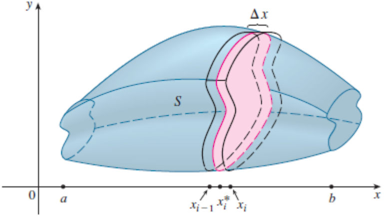
Figura: 

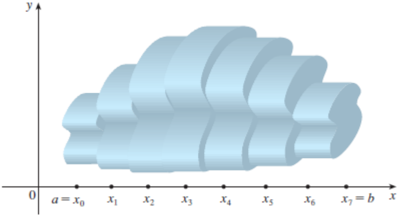


$$V = A(x_i^*)\Delta x_i \tag{3}$$

$$V = \lim_{n\to\infty}\sum_{i=1}^{\infty} V_i = \lim_{n\to\infty}\sum_{i=1}^{\infty} A(x_i^*)\Delta x_i \tag{4}$$

$$V = \int_a^b A(x)dx \tag{5}$$





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Exemplo - Volume da Esfera

Mostre que o volume da esfera de raio r é $\frac{4}{3}\pi r^3$.

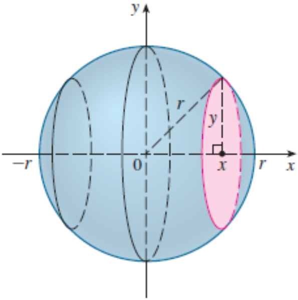

Figura:

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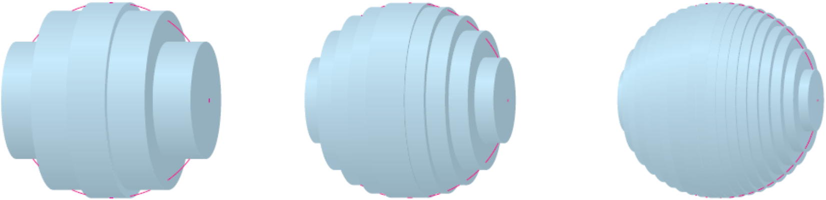


(a) Using 5 disks, $V \approx 4.2726$

(b) Using 10 disks, $V \approx 4.2097$

(c) Using 20 disks, $V \approx 4.1940$

Exemplo 2

Calcule o volume da região obtida rotacionando a área delimitada pela curva $y = \sqrt{x}$ com $0 \leq x \leq 1$.

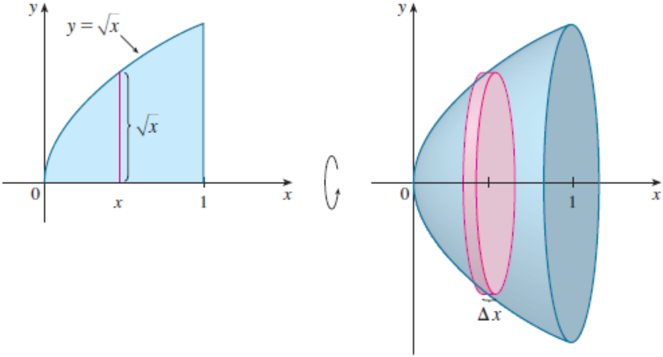
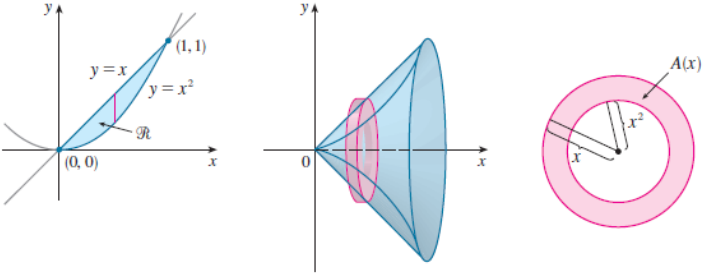


Figura:

Exemplo 3

A região delimitada pelas curvas $y = x$ e $y = x^2$ é rotacionada em torno do eixo x. Determine seu volume.

Figura:



Qual o volume do sólido obtido rotacionando a região em torno do eixo y?

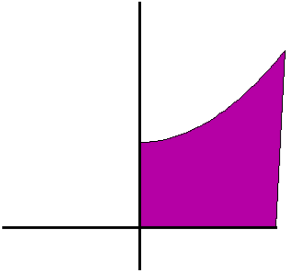
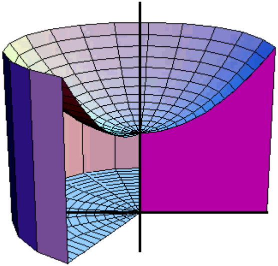
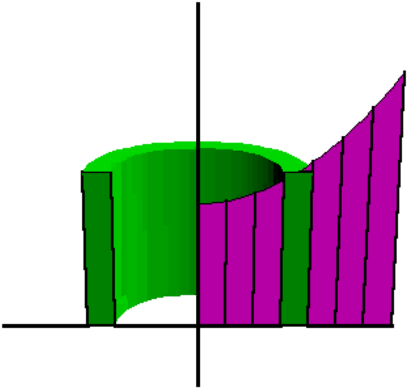
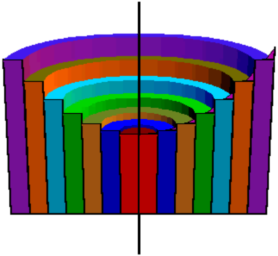


Figura:

Figura: 





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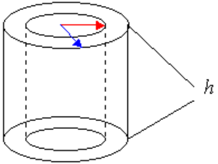
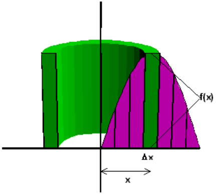
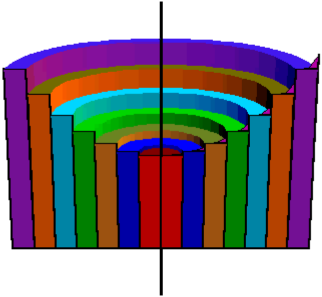




$$
\begin{aligned}
V &= \pi(R^2 - r^2)h \\
&= 2\pi\frac{(R+r)}{2}(R-r)h \\
&= 2\pi r^* \Delta r h
\end{aligned}
$$



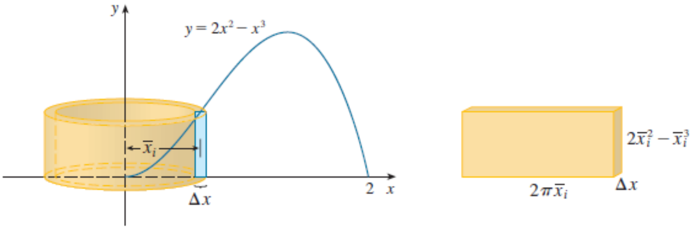
$$V = \sum_{i=1} 2\pi x_i^* \Delta x_i f(x)$$

$$V = \int_a^b 2\pi x f(x) dx.$$

Exemplos

Calcule o volume do sólido obtido rotacionando em torno do eixo y a região delimitada por $y = 2x^{2-x^3}$ e pelo eixo x.

Figura:



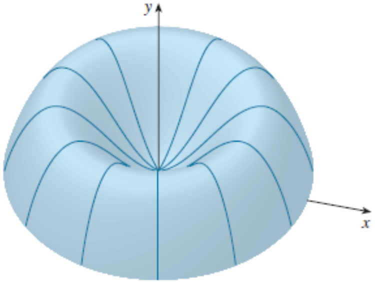


Figura: