

Introdução a Teoria dos Grupos
– MAT 113 – Pós Mat – UFABC-
QS2020.2

LATTICE DE SUBGRUPOS DE UM GRUPO:

Ex: $\frac{\mathbb{Z}}{2\mathbb{Z}} = \langle 1 \rangle$

|
 $\langle 2 \rangle = \{0\}$

$\frac{\mathbb{Z}}{8\mathbb{Z}} = \langle 1 \rangle$

|
 $\langle 2 \rangle$

|
 $\langle 4 \rangle$

|
 $\langle 8 \rangle = \{0\}$

$\frac{\mathbb{Z}}{4\mathbb{Z}} = \langle 1 \rangle$

|
 $\langle 2 \rangle$

|
 $\langle 4 \rangle = \{0\}$

Obs; $\langle 1 \rangle = \langle 3 \rangle$.

Obs: $\langle 1 \rangle = \langle 3 \rangle = \langle 5 \rangle = \langle 7 \rangle$.

LATTICE DE SUBGRUPOS DE UM GRUPO:

Ex: $p \in \mathbb{Z}$, p primo

$$\frac{\mathbb{Z}}{p^n \mathbb{Z}} = \langle 1 \rangle$$

$$\langle p \rangle$$

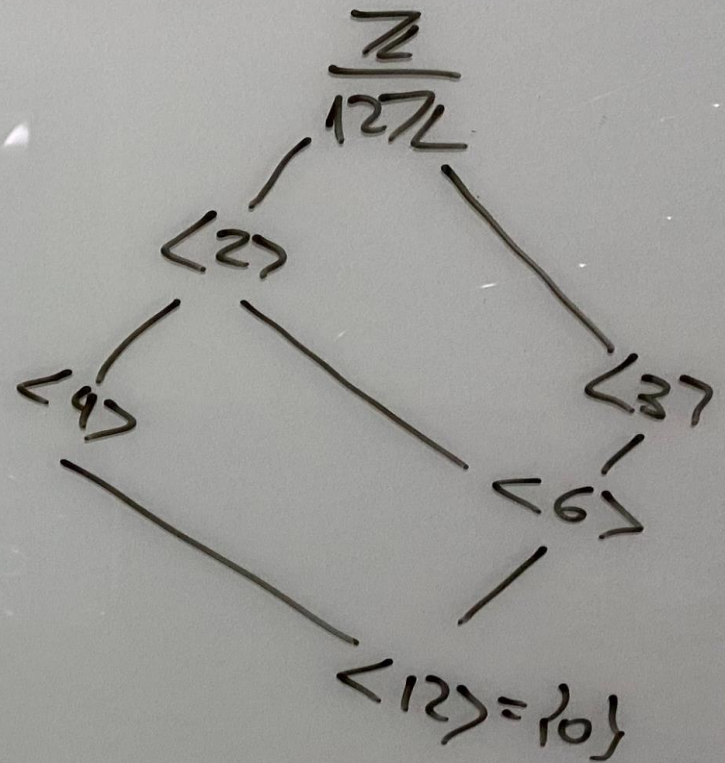
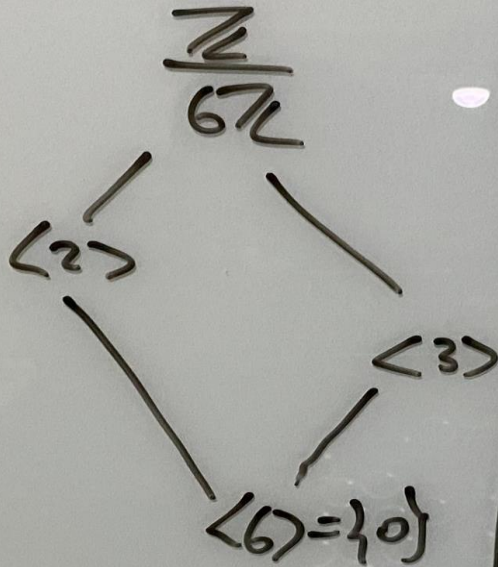
$$\langle p^2 \rangle$$

$$\langle p^3 \rangle$$

...

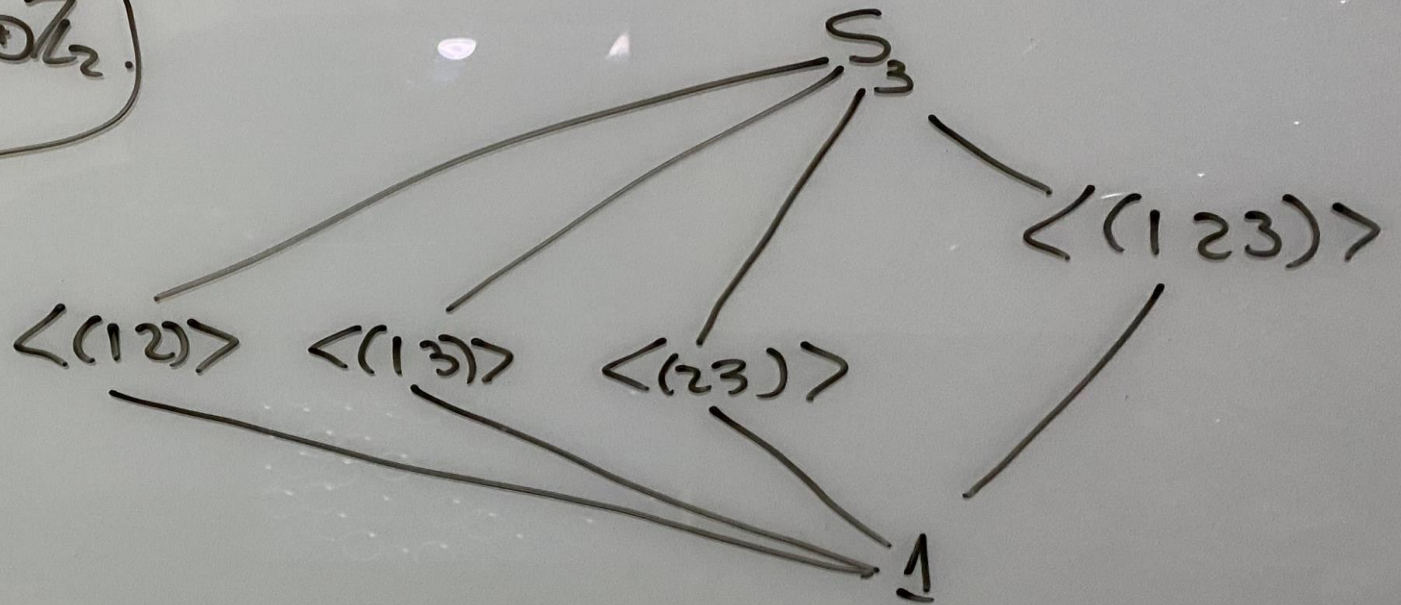
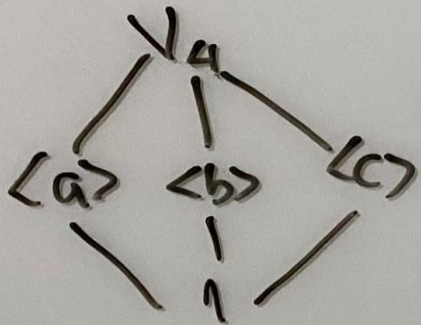
$$\langle p^{n-1} \rangle$$

$$\langle p^n \rangle = \{0\}$$



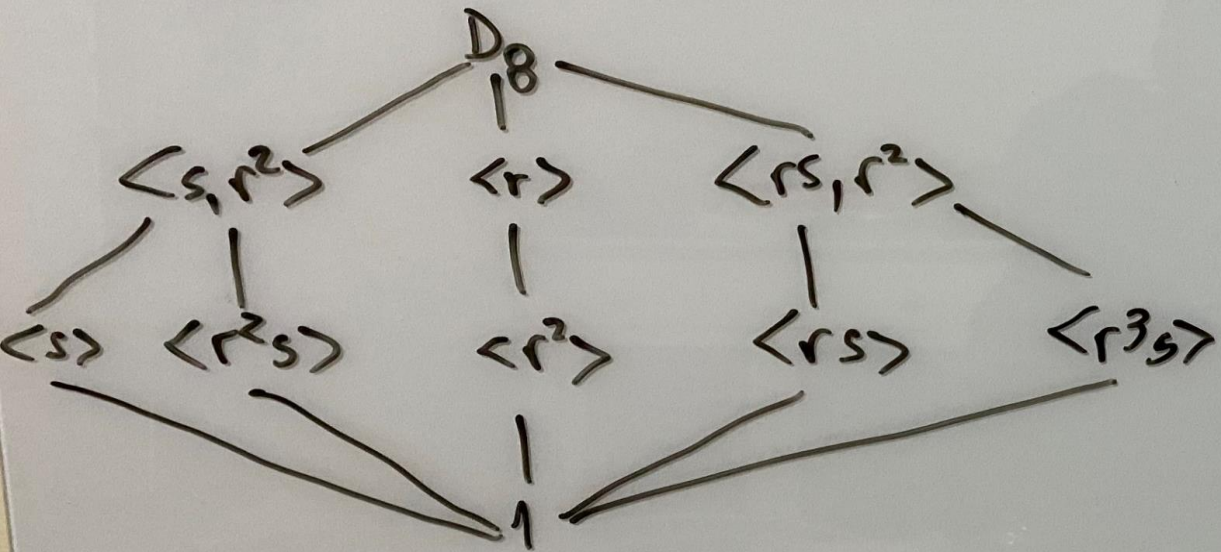
LATTICE DE SUBGRUPOS DE UM GRUPO:

GRUPO 4 DE Klein $V \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$.
 $V = \{e, a, b, c\}$ $a^2 = b^2 = c^2 = e$.



LATTICE DE SUBGRUPOS DE UM GRUPO

$$D_8 = \langle r, s \mid r^4 = 1, s^2 = 1, rs = sr^{-1} \rangle$$



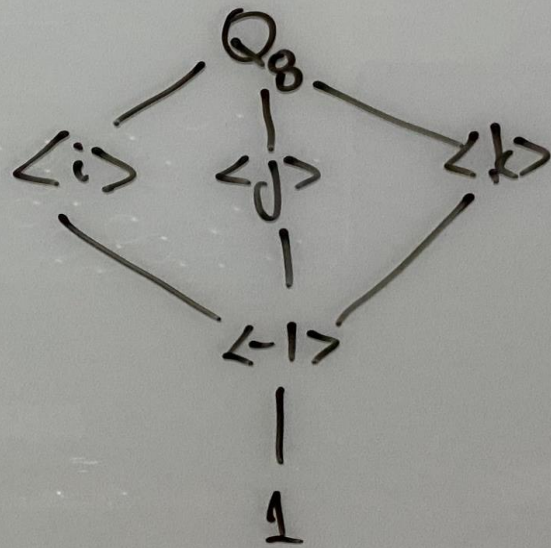
$$Q_8 = \{ \pm 1, \pm i, \pm j, \pm k \}$$

$$i^2 = j^2 = k^2 = -1,$$

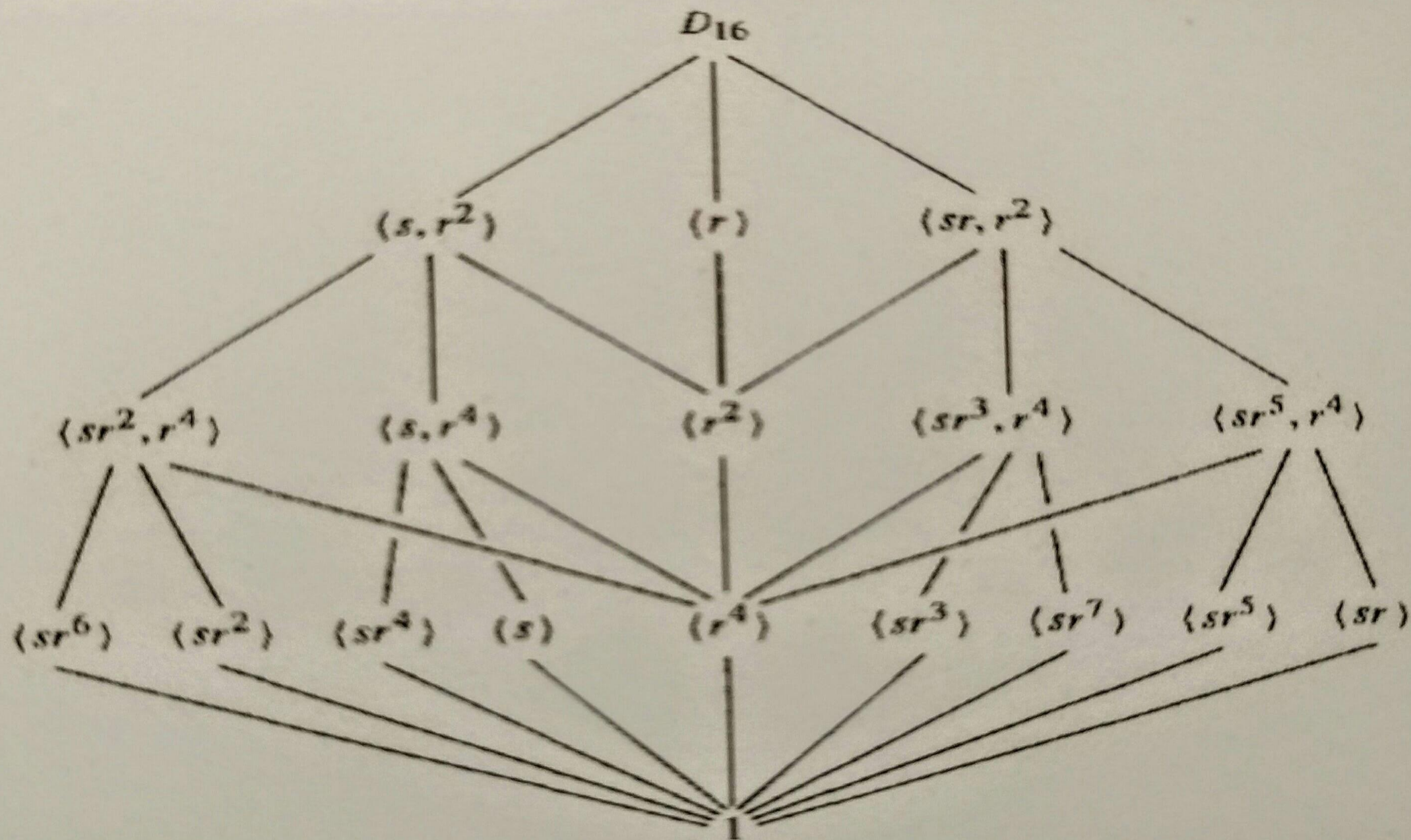
$$ij = k = -ji$$

$$jk = i = -kj$$

$$ki = j = -ik$$



(6) The lattice of D_{16} is not a planar graph (cannot be drawn on a plane without lines crossing). One way of drawing it is



- O lattice de subgrupos do grupo $D_{\{16\}}$ foi tirado do livro Abstract Algebra, 3rd Ed., Dummit and Foote.