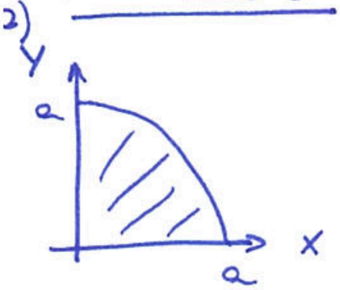
Questão 1

a)



$$\int_0^a \int_0^{\sqrt{a^2-x^2}} dy\,dx = \int_0^{\pi/2} \int_0^a r\,dr\,d\theta$$

$$= \int_0^{\pi/2} 1 \cdot d\theta \cdot \int_0^a r\,dr$$

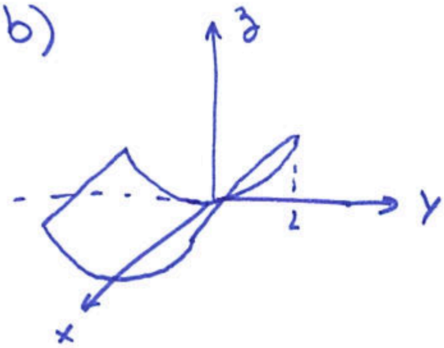
$$= \frac{\pi}{2} \cdot \left(\frac{r^2}{2}\right)\Bigg|_0^a = \boxed{\frac{a^2\pi}{4}}$$

$y = \sqrt{a^2 - x^2}$

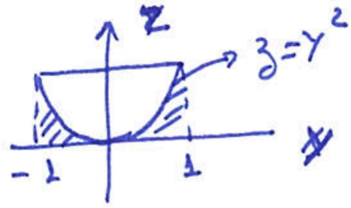
$y^2 + x^2 = a^2$

$R = \{ (r,\theta) \mid 0 \leq r \leq a,\ 0 \leq \theta \leq \pi/2 \}$

b)



Tipo 2 : D contida no plano ~~x~~yz



$-1 \leq y \leq 1$

$0 \leq z \leq y^2$

$D = \{ (\cancel{x}, z) \mid -1 \leq y \leq 1,\ 0 \leq z \leq y^2 \}$

• x vai de $x = 0$ até $x = 1$

$$V = \iiint_R dV = \int_0^1 \int_{-1}^1 \int_0^{y^2} dz\,dy\,dx = \int_0^1 \int_{-1}^1 y^2\,dy\,dx$$

$$= \int_0^1 \frac{y^3}{3}\Bigg|_{-1}^1 dx = \int_0^1 \frac{1}{3} - \frac{(-1)}{3}\,dx = \frac{2}{3}\int_0^1 1\,dx$$

$$= \boxed{\frac{2}{3}}$$

Questão 2

$f(x,y,z) = xyz$

$R = \{(x,y,z) \mid x \geq 0, y \geq 0, z \geq 0, x + 2y + 2z \leq 108\}$

$$\begin{cases} f_x = yz = 0 \\ f_y = xz = 0 \\ f_z = xy = 0 \end{cases} \Rightarrow$$

Pontos críticos:

$(0,0,z)$, $(x,0,0)$, $(0,y,0)$

Nos pontos críticos temos $f = 0$.
(mínimo absoluto)

$g(x,y,z) = x + 2y + 2z = 108$

Multiplicadores de Lagrange:

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y,z) = 108 \end{cases} \Rightarrow \begin{cases} yz = \lambda \cdot 1 & (3) \\ xz = \lambda \cdot 2 & (1) \\ xy = \lambda \cdot 2 & (2) \\ x + 2y + 2z = 108 & (4) \end{cases} \underset{\substack{x \neq 0 \\ (1) \div (2)}}{\Rightarrow} y = z$$

$(3) \Rightarrow z^2 = \lambda \Rightarrow z = \sqrt{\lambda} \Rightarrow y = \sqrt{\lambda}$

$(1) \Rightarrow x\sqrt{\lambda} = 2\lambda \Rightarrow x = 2\sqrt{\lambda}$

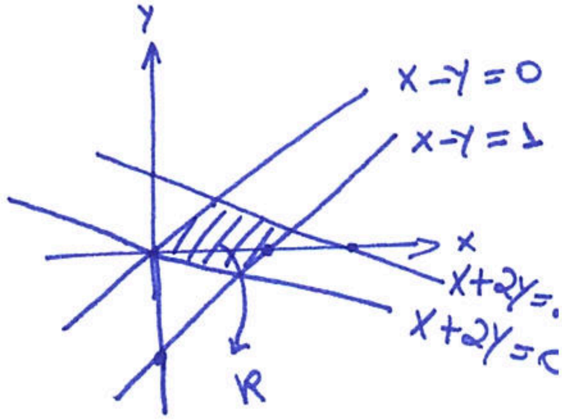
$(4) \Rightarrow 2\sqrt{\lambda} + 2\sqrt{\lambda} + 2\sqrt{\lambda} = 108 \Rightarrow \sqrt{\lambda} = 18$

$$\boxed{\begin{array}{c} x = 36 \\ \hline y = 18 \\ \hline z = 18 \end{array}}$$

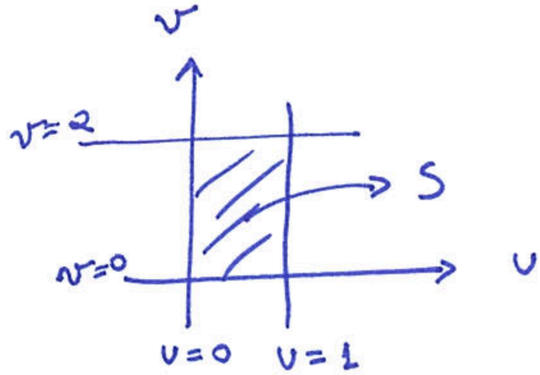
$$\boxed{V = 36 \cdot 18 \cdot 18}$$

Questão 3

$$\begin{cases} v = x + 2y \\ u = x - y \end{cases} \Rightarrow \begin{cases} y = \dfrac{v-u}{3} \\ x = \dfrac{2u+v}{3} \end{cases}$$



- $x - y = 0 \Rightarrow u = 0$
- $x - y = 1 \Rightarrow u = 1$
- $x + 2y = 0 \Rightarrow v = 0$
- $x + 2y = 2 \Rightarrow v = 2$



$$T(u,v) = (x,y) = \left(\frac{2u+v}{3}, \frac{v-u}{3} \right)$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 2/3 & 1/3 \\ -1/3 & 1/3 \end{vmatrix} = \frac{2}{9} + \frac{1}{9} = \frac{1}{3}$$

$$\iint_R \frac{x+2y}{\cos(x-y)}\, dA = \iint_S \frac{v}{\cos u} \cdot \left|\frac{1}{3}\right| \cdot dA = \int_0^2 \int_0^1 \frac{v}{\cos u} \cdot \frac{1}{3}\, du\, dv$$

$$= \frac{1}{3} \int_0^2 v\, dv \cdot \int_0^1 \sec u\, du = \frac{1}{3} \cdot \frac{v^2}{2}\Big|_0^2 \cdot \ln|\sec u + \operatorname{tg} u|\Big|_0^1$$

$$= \frac{1}{3} \cdot 2 \cdot \left(\ln|\sec 1 + \operatorname{tg} 1| - \ln|\underbrace{\sec 0}_{1/\cos 0 = 1} + \operatorname{tg} 0| \right)$$

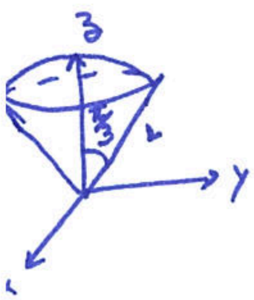
$$= \boxed{\frac{2}{3} \left(\ln|\sec 1 + \operatorname{tg} 1| \right)}$$

Questão 4

$$Vol(E) = \iiint_E 1\, dV = \int_0^{2\pi}\int_0^{\pi/3}\int_0^{1} \rho^2 \operatorname{sen}\phi \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi} d\theta \cdot \int_0^{1} \rho^2 d\rho \cdot \int_0^{\pi/3} \operatorname{sen}\phi\, d\phi$$

$$= 2\pi \cdot \frac{\rho^3}{3}\Big|_0^1 \cdot (-\cos\phi)\Big|_0^{\pi/3} = \frac{2\pi}{3}\cdot\left(-\frac{1}{2}+1\right) = \frac{\pi}{3}$$



$$\bar{z} = \frac{3}{\pi}\cdot\iiint_E z\, dV = \frac{3}{\pi}\int_0^{\pi/3}\int_0^{2\pi}\int_0^{1} \underbrace{\rho\cos\phi}_{z}\, \rho^2 \operatorname{sen}\phi \, d\rho\, d\theta\, d\phi$$

$$= \frac{3}{\pi}\cdot\int_0^{2\pi} d\theta \cdot \int_0^{\pi/3}\int_0^{1} \rho^3 \cos\phi \operatorname{sen}\phi\, d\rho\, d\phi$$

$$= \frac{3}{\pi}\cdot 2\pi \cdot \int_0^{\pi/3}\left(\frac{\rho^4}{4}\cos\phi\operatorname{sen}\phi\right)\Big|_0^1 d\phi$$

$$= \frac{6}{4}\int_0^{\pi/3} \operatorname{sen}\phi\cos\phi\, d\phi \quad = \frac{3}{2}\int_0^{\sqrt{3}/2} u\, du$$

$u = \operatorname{sen}\phi$

$du = \cos\phi\, d\phi$

$\phi = 0 \Rightarrow u = 0$

$\phi = \pi/3 \Rightarrow u = \sqrt{3}/2$

$$= \frac{3}{2}\,\frac{u^2}{2}\Big|_0^{\sqrt{3}/2}$$

$$= \frac{3}{2}\cdot\frac{3/4}{2} = \frac{3}{2}\cdot\frac{3}{8} = \boxed{\frac{9}{16}}$$

Questão 5

$$\begin{cases} 0 \leq \rho \leq 4\cos\phi \\ 0 \leq \theta \leq 2\pi \\ 0 \leq \phi \leq \pi/3 \end{cases}$$

$$V = \iiint_R 1 \, dV$$

$$= \int_0^{\pi/3} \int_0^{2\pi} \int_0^{4\cos\phi} \rho^2 \operatorname{sen}\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_0^{\pi/3} \int_0^{2\pi} \frac{\rho^3}{3} \operatorname{sen}\phi \Big|_0^{4\cos\phi} d\theta \, d\phi$$

$$= \int_0^{\pi/3} \int_0^{2\pi} \frac{4^3\cos^3\phi}{3} \operatorname{sen}\phi \, d\theta \, d\phi$$

$$= 2\pi \cdot \frac{4^3}{3} \int_0^{\pi/3} \cos^3\phi \operatorname{sen}\phi \, d\phi \qquad \begin{cases} u = \cos\phi \\ du = -\operatorname{sen}\phi \\ \phi = 0 \Rightarrow u = 1 \\ \phi = \pi/3 \Rightarrow u = \frac{1}{2} \end{cases}$$

$$= \frac{2\pi \cdot 4^3}{3} \cdot \int_1^{1/2} u^3 (-du)$$

$$= -\frac{2\pi \cdot 4^3}{3} \cdot \frac{u^4}{4} \Big|_1^{1/2}$$

$$= -\frac{2\pi \cdot 16}{3} \left(\left(\frac{1}{2}\right)^4 - 1^4 \right) = \boxed{10\pi}$$