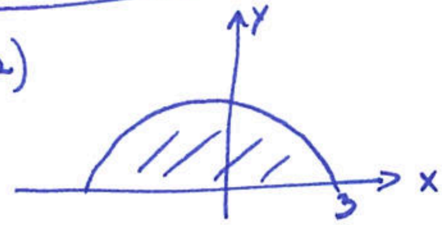
Questão 1

a)



$$\iint_R \operatorname{sen}(x^2+y^2)\,dA$$

$R=\{(r,\theta)\mid 0\le r\le 3,\ 0\le \theta\le \pi\}$

$$\begin{cases} x = r\cos\theta \\ y = r\operatorname{sen}\theta \end{cases}$$

$$= \int_0^{\pi}\int_0^{3} \operatorname{sen}(r^2)\, r\, dr\, d\theta$$

$$= \int_0^{\pi} d\theta \int_0^{3} \operatorname{sen}(r^2).\, r\, dr =$$

$u = r^2$

$du = 2r\,dr$

$r=0 \Rightarrow u=0$

$r=3 \Rightarrow u=9$

$$= \pi \cdot \int_0^{9} \operatorname{sen}(u)\cdot \frac{du}{2} = \frac{\pi}{2}\left(-\cos u\right)\Big|_0^9$$

$$= \pi/2\,(-\cos 9 + \cos 0) = \boxed{\pi/2\,(1-\cos 9)}$$

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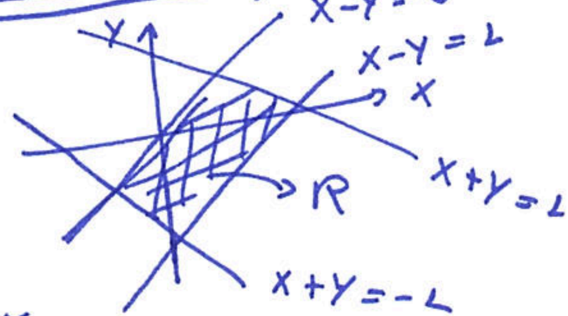
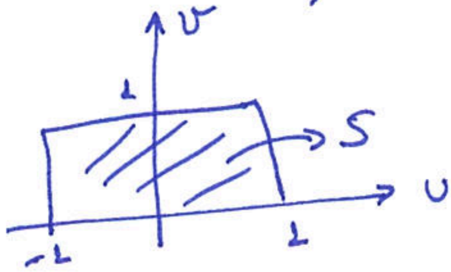
b) $\begin{cases} u = x+y \\ v = x-y \end{cases} \Rightarrow \begin{cases} x = \dfrac{u+v}{2} \\ y = \dfrac{u-v}{2} \end{cases}$



- $x+y=-1 \Rightarrow u=-1$
- $x+y=1 \Rightarrow u=1$
- $x-y=0 \Rightarrow v=0$
- $x-y=1 \Rightarrow v=1$



$$T(u,v)=(x,y)=\left(\frac{u+v}{2},\ \frac{u-v}{2}\right)$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{vmatrix} = -\frac{1}{4}-\frac{1}{4} = -\frac{1}{2}$$

$$\iint_R xy\,dx\,dy = \iint_S \frac{u+v}{2}\cdot\frac{u-v}{2}\cdot\left|-\frac{1}{2}\right|.\,du\,dv =$$

$$= \int_0^1 \int_{-1}^{1} \left(\frac{(u^2 - v^2)}{4 \cdot 2} \right) du\, dv = \frac{1}{8} \int_0^1 \int_{-1}^{1} (u^2 - v^2) du\, dv$$

$$= \frac{1}{8} \int_0^1 \left(\frac{u^3}{3} - v^2 u \right) \Big|_{u=-1}^{u=1} dv$$

$$= \frac{1}{8} \int_0^1 \left(\frac{1}{3} - v^2 + \frac{1}{3} - v^2 \right) dv = \frac{1}{8} \int_0^1 \left(\frac{2}{3} - 2v^2 \right) dv$$

$$= \frac{1}{8} \left(\frac{2}{3} v - \frac{2v^3}{3} \right) \Big|_0^1 = \frac{1}{4} \left(\frac{1}{3} - \frac{1}{3} \right) = \boxed{0.}$$

Questão 2

$$f(x,y,z) = xyz$$

$$R = \{(x,y,z) \mid x \geq 0, y \geq 0, z \geq 0, x + 2y + 2z \leq 108\}$$

$$\begin{cases} f_x = yz = 0 \\ f_y = xz = 0 \\ f_z = xy = 0 \end{cases} \Rightarrow$$

Pontos críticos:

$(0,0,z)$, $(x,0,0)$, $(0,y,0)$

Nos pontos críticos temos $f = 0$.
(mínimo absoluto)

$$g(x,y,z) = x + 2y + 2z = 108$$

Multiplicadores de Lagrange:

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y,z) = 108 \end{cases} \Rightarrow \begin{cases} yz = \lambda \cdot 1 & (3) \\ xz = \lambda \cdot 2 & (1) \\ xy = \lambda \cdot 2 & (2) \\ x + 2y + 2z = 108 & (4) \end{cases} \xrightarrow[x \neq 0]{} y = z$$

$(1) = (2)$

$$(3) \Rightarrow z^2 = \lambda \Rightarrow z = \sqrt{\lambda} \Rightarrow y = \sqrt{\lambda}$$

$$(1) \Rightarrow x\sqrt{\lambda} = 2\lambda \Rightarrow x = 2\sqrt{\lambda}$$

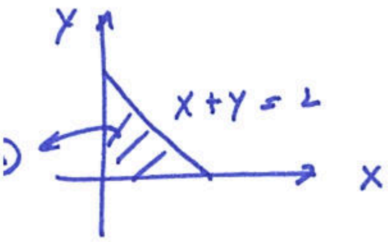
$$(4) \Rightarrow 2\sqrt{\lambda} + 2\sqrt{\lambda} + 2\sqrt{\lambda} = 108 \Rightarrow \sqrt{\lambda} = 18$$

$$\boxed{\begin{array}{c} x = 36 \\ \hline y = 18 \\ \hline z = 18 \end{array}}$$

$$\boxed{V = 36 \cdot 18 \cdot 18}$$

Questão 3

Tipo 1: D contida no plano xy



$0 \le x \le 1$

$0 \le y \le 1-x$

$D = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1-x\}$

z vai de 0 até o plano $x+y+z=1$

$\downarrow$

$(z = 1-x-y)$

$$m = \iiint_T xyz \, dV = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz \, dz \, dy \, dx =$$

$$= \int_0^1 \int_0^{1-x} \frac{xy(1-x-y)^2}{2} \, dy \, dx = -\int_0^1 \int_{1-x}^0 \frac{x(1-x-u)u^2}{2} \, du \, dx$$

$$\begin{cases} 1-x-y = u \Rightarrow y = 1-x-u \\ du = -dy \\ y = 0 \Rightarrow u = 1-x \\ y = 1-x \Rightarrow u = 0 \end{cases}$$

$$= -\frac{1}{2} \int_0^1 x \int_{1-x}^0 (1-x)u^2 - u^3 \, du \, dx$$

$$= -\frac{1}{2} \int_0^1 x \left[(1-x)\frac{u^3}{3} - \frac{u^4}{4} \right] \Bigg|_{1-x}^0 dx$$

$$= -\frac{1}{2} \int_0^1 x \left(-\frac{(1-x)^4}{3} + \frac{(1-x)^4}{3} \right) dx = \frac{1}{2} \int_0^1 x \cdot \frac{(1-x)^4}{12} \, dx$$

$$= \frac{1}{24} \int_0^1 x(1-x)^4 \, dx$$

$$\begin{cases} v = 1-x \Rightarrow x = 1-v \Rightarrow dv = -dx \\ x = 0 \Rightarrow v = 1 \\ x = 1 \Rightarrow v = 0 \end{cases}$$

$$= -\frac{1}{24} \int_1^0 (1-v) v^4 \, dv = -\frac{1}{24} \left(\frac{v^5}{5} - \frac{v^5}{6} \right) \Bigg|_0^1 =$$

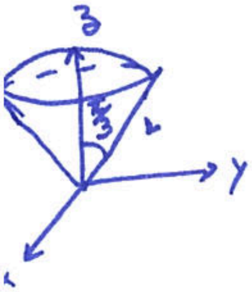
$$= \boxed{\frac{1}{24} \left(\frac{1}{5} - \frac{1}{6} \right)}$$

Questão 4

$$Vol(E) = \iiint_E 1\, dV = \int_0^{2\pi}\int_0^{\pi/3}\int_0^{1} \rho^2 \operatorname{sen}\phi \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi} d\theta \cdot \int_0^{1} \rho^2 d\rho \cdot \int_0^{\pi/3} \operatorname{sen}\phi\, d\phi$$

$$= 2\pi \cdot \frac{\rho^3}{3}\Big|_0^1 \cdot (-\cos\phi)\Big|_0^{\pi/3} = \frac{2\pi}{3}\cdot\left(-\frac{1}{2}+1\right) = \frac{\pi}{3}$$



$$\bar{z} = \frac{3}{\pi}\cdot\iiint_E z\, dV = \frac{3}{\pi}\int_0^{\pi/3}\int_0^{2\pi}\int_0^{1} \underbrace{\rho\cos\phi}_{z}\, \rho^2 \operatorname{sen}\phi\, d\rho\, d\theta\, d\phi$$

$$= \frac{3}{\pi}\cdot\int_0^{2\pi} d\theta \cdot \int_0^{\pi/3}\int_0^{1} \rho^3 \cos\phi \operatorname{sen}\phi\, d\rho\, d\phi$$

$$= \frac{3}{\pi}\cdot 2\pi \cdot \int_0^{\pi/3}\left(\frac{\rho^4}{4}\cos\phi\operatorname{sen}\phi\right)\Big|_0^1 d\phi$$

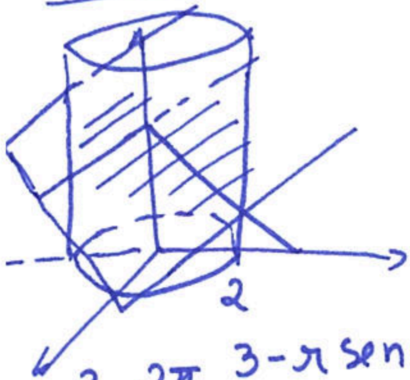
$$= \frac{6}{4}\int_0^{\pi/3} \operatorname{sen}\phi\cos\phi\, d\phi \quad = \frac{3}{2}\int_0^{\sqrt{3}/2} u\, du$$

$u = \operatorname{sen}\phi$
$du = \cos\phi\, d\phi$
$\phi = 0 \Rightarrow u = 0$
$\phi = \pi/3 \Rightarrow u = \sqrt{3}/2$

$$= \frac{3}{2}\,\frac{u^2}{2}\Big|_0^{\sqrt{3}/2}$$

$$= \frac{3}{2}\cdot\frac{3/4}{2} = \frac{3}{2}\cdot\frac{3}{8} = \boxed{\frac{9}{16}}$$

Questão 5



Coordenadas cilíndricas

$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \\ z = z \end{cases} \qquad \begin{aligned} &0 \le \theta \le 2\pi \\ &0 \le r \le 2 \\ &0 \le z \le 3 - y = 3 - r\,\text{sen}\,\theta \end{aligned}$$

$$\int_0^2 \int_0^{2\pi} \int_0^{3 - r\,\text{sen}\,\theta} r \, dz \, d\theta \, dr$$

$$= \int_0^2 \int_0^{2\pi} r(3 - r\,\text{sen}\,\theta) \, d\theta \, dr$$

$$= \int_0^2 \int_0^{2\pi} (3r - r^2\,\text{sen}\,\theta) \, d\theta \, dr = \int_0^{2\pi} \left(\frac{3r^2}{2} - \frac{r^3}{3}\,\text{sen}\,\theta \right) \Bigg|_{r=0}^{r=2} d\theta$$

$$= \int_0^{2\pi} \left(6 - \frac{8}{3}\,\text{sen}\,\theta \right) d\theta = \left(6\theta - \frac{8}{3}\cos\theta \right) \Bigg|_{\theta=0}^{\theta=2\pi}$$

$$= \boxed{12\pi}$$