Questão 2

(a) $C_1 : \underline{x=0}$

$f(0,y) = \dfrac{0}{y^3} = 0 \Rightarrow f(x,y) \to 0$ ao longo de C_1

$C_2 : \underline{x=y}$

$f(x,x) = \dfrac{x^2}{2x^3} = \dfrac{1}{2x} \Rightarrow f(x,y) \to \infty$ ao longo de C_2

Logo $\nexists \lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{y(x^2+y^2)}$.

b) $z_{st} = \dfrac{\partial}{\partial t}\left(\dfrac{\partial z}{\partial s}\right)$ $\quad x_s = e^s \cos t \quad x_t = -e^s \operatorname{sen} t$

$\quad y_s = e^s \operatorname{sen} t \quad y_t = e^s \cos t$

$z_s = z_x \cdot x_s + z_y \cdot y_s$

$= z_x (e^s \cos t) + z_y \cdot (e^s \operatorname{sen} t)$

$z_{st} = \dfrac{\partial}{\partial t}\left(z_x \cdot (e^s \cos t) + z_y (e^s \operatorname{sen} t)\right)$

$= \dfrac{\partial z_x}{\partial t} \cdot (e^s \cos t) + z_x \cdot \dfrac{\partial}{\partial t}(e^s \cos t) + \dfrac{\partial z_y}{\partial t}(e^s \operatorname{sen} t) +$

$+ z_y \cdot \dfrac{\partial}{\partial t}(e^s \operatorname{sen} t)$

$= (z_{xx} \cdot x_t + z_{xy} \cdot y_t)(e^s \cos t) + z_x \cdot (-e^s \operatorname{sen} t) +$

$+ (z_{yx} \cdot x_t + z_{yy} \cdot y_t)(e^s \operatorname{sen} t) + z_y (e^s \cos t)$

$= [z_{xx} \cdot (-e^s \operatorname{sen} t) + z_{xy} \cdot (e^s \cos t)](e^s \cos t) + z_x \cdot (-e^s \operatorname{sen} t) +$

$+ [z_{yx} \cdot (-e^s \operatorname{sen} t) + z_{yy} \cdot (e^s \cos t)] \cdot (e^s \operatorname{sen} t) + z_y (e^s \cos t)$

Questão 2

a) Calcular $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$.

C_1 : $x=0$

$f(0,y) = \frac{0}{y^2} = 0 \Rightarrow f(x,y) \to 0$ ao longo de C_1

C_2 : $x=y$

$f(x,x) = \frac{x^2}{2x^2} = \frac{1}{2} \Rightarrow f(x,y) \to \frac{1}{2}$ ao longo de C_2

Logo $\nexists \lim_{(x,y)\to(0,0)} f(x,y)$. Portanto, f não é contínua em $(0,0)$.

b) • $(x,y) \neq (0,0)$

$$f_x(x,y) = \frac{y(x^2+y^2) - xy(2x)}{(x^2+y^2)^2} = \frac{y(y^2-x^2)}{(x^2+y^2)^2}$$

$$f_y(x,y) = \frac{x(x^2+y^2) - xy(2y)}{(x^2+y^2)^2} = \frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

• $(x,y) = (0,0)$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

$$f_y(0,0) = \lim_{h\to 0} \frac{f(0,0+h)-f(0,0)}{h} = 0$$

$$f_x(x,y) = \begin{cases} \frac{y(y^2-x^2)}{(x^2+y^2)^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}, \quad f_y(x,y) = \begin{cases} \frac{x(x^2-y^2)}{(x^2+y^2)^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

c) Não, pois f não é contínua em $(0,0)$.

Questão 3

a) $\|\vec{v}\| = \sqrt{(1)^2+(1)^2+(-1)^2} = \sqrt{3}$

$\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{1}{\sqrt{3}}(1, 1, -1)$

$\nabla V(x,y,z) = (2x - 3y + yz, -3x + xz, xy)$

$\nabla V(3,4,5) = (2\cdot3 - 3\cdot4 + 4\cdot5, -3\cdot3 + 3\cdot5, 3\cdot4)$
$= (6 - 12 + 20, -9 + 15, 12)$
$= (14, 6, 12)$

$D_u V(3,4,5) = \nabla V(3,4,5) \cdot \vec{u}$
$= (14, 6, 12) \cdot \dfrac{1}{\sqrt{3}}(1, 1, -1)$
$= \dfrac{1}{\sqrt{3}}(14 + 6 - 12) = \dfrac{8}{\sqrt{3}}$

(b) $\nabla V(3,4,5) = (14, 6, 12)$

(c) $\|\nabla(3,4,5)\| = \|(14,6,12)\| = \sqrt{14^2+6^2+12^2}$

Questão 4

(a) $Dom f = \mathbb{R}^2$, $Im f = \mathbb{R}^+_*$

(b) curva de nível $K = \{(x,y) \in \mathbb{R}^2 \mid f(x,y) = K\}$
$= \{(x,y) \in \mathbb{R}^2 \mid e^{x+y} = K\}$
$= \{(x,y) \in \mathbb{R}^2 \mid x+y = \ln K\}$, $\underline{K > 0}$

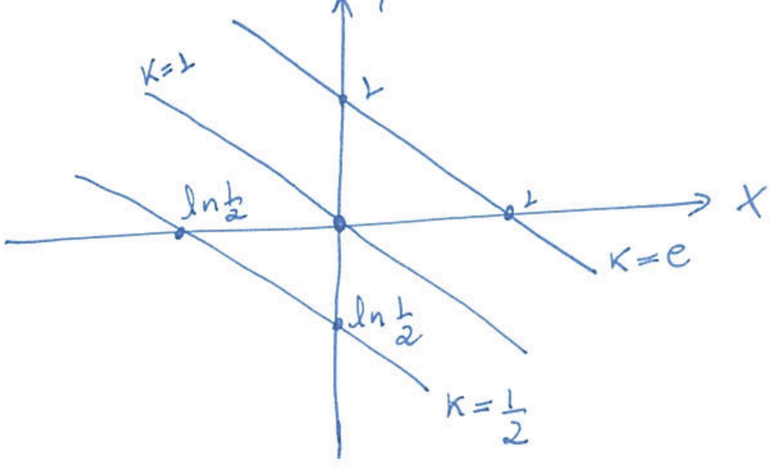
$\boxed{y = -x + \ln K}$

$K > 1 \Rightarrow \ln K > 0$
$K = 1 \Rightarrow \ln K = 0$
$0 < K < 1 \Rightarrow \ln K < 0$

$\underline{K=1}$ $\quad y = -x$

$\underline{K=e}$ $\quad y = -x + 1$

$\underline{K=\frac{1}{2}}$ $\quad y = -x + \underbrace{\ln\frac{1}{2}}_{<0}$



Questão 5

Plano tangente ao gráfico de f em $(a, b, f(a,b))$:

$$\boxed{z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b)}$$

$$f_x(x,y) = \psi\left(\frac{x}{y}\right) + x \cdot \psi'\left(\frac{x}{y}\right) \cdot \frac{1}{y} = \psi\left(\frac{x}{y}\right) + \frac{x}{y}\psi'\left(\frac{x}{y}\right)$$

$$f_y(x,y) = x \cdot \psi'\left(\frac{x}{y}\right) \cdot \left(-\frac{x}{y^2}\right) = -\frac{x^2}{y^2}\psi'\left(\frac{x}{y}\right)$$

$(x,y) = (a,b)$

$$f(a,b) = a\,\psi\left(\frac{a}{b}\right)$$

$$f_x(a,b) = \psi\left(\frac{a}{b}\right) + \frac{a}{b}\psi'\left(\frac{a}{b}\right)$$

$$f_y(a,b) = -\frac{a^2}{b^2}\psi'\left(\frac{a}{b}\right)$$

Equação do plano tangente:

$$z - \left[a \cdot \psi\left(\frac{a}{b}\right)\right] = \left[\psi\left(\frac{a}{b}\right) + \frac{a}{b}\psi'\left(\frac{a}{b}\right)\right](x-a) + \left[-\frac{a^2}{b^2}\psi'\left(\frac{a}{b}\right)\right](y-b).$$

Se $x = y = 0$, o lado ~~esqu~~ direito da equação é:

$$\left[\psi\left(\frac{a}{b}\right) + \frac{a}{b}\psi'\left(\frac{a}{b}\right)\right](-a) + \left[-\frac{a^2}{b^2}\psi'\left(\frac{a}{b}\right)\right](-b) =$$

$$= -a\,\psi\left(\frac{a}{b}\right) - \frac{a^2}{b}\psi'\left(\frac{a}{b}\right) + \frac{a^2}{b}\psi'\left(\frac{a}{b}\right) = -a\,\psi\left(\frac{a}{b}\right)$$

O lado esquerdo da equação é $z - \left[a \cdot \psi\left(\frac{a}{b}\right)\right]$.

Se $z = 0$, então o lado esquerdo da equação é $-a \cdot \psi\left(\frac{a}{b}\right)$.

Logo $(x,y,z) = (0,0,0)$ satisfaz a equação do plano tangente. Concluímos que ele passa pela origem.