Questão 1

(a) C_1 : $x = 0$

$f(0,y) = \frac{0}{y^4} = 0 \Rightarrow f(x,y) \to 0$ ao longo de C_1

C_2 : $x = \cancel{y}^2$

$f(y^2,y) = \frac{(y^2)^2}{(y^2)^2+y^4} = \frac{y^4}{2y^4} = \frac{1}{2} \Rightarrow f(x,y) \to \frac{1}{2}$ ao longo de C_2.

Logo $\nexists \lim\limits_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$.

(b) $z_{st} = \frac{\partial}{\partial t}\left(\frac{\partial z}{\partial s}\right)$ $\quad x_s = e^s \cos t \quad x_t = -e^s \operatorname{sen} t$

$y_s = e^s \operatorname{sen} t \quad y_t = e^s \cos t$

$z_s = z_x \cdot x_s + z_y \cdot y_s$

$= z_x \cdot (e^s \cos t) + z_y \cdot (e^s \operatorname{sen} t)$

$z_{st} = \frac{\partial}{\partial t}\left(z_x \cdot (e^s \cos t) + z_y \cdot (e^s \operatorname{sen} t)\right)$

$= \frac{\partial}{\partial t} z_x \cdot (e^s \cos t) + z_x \cdot \frac{\partial}{\partial t}(e^s \cos t) + \frac{\partial}{\partial t} z_y (e^s \operatorname{sen} t) +$

$+ z_y \cdot \frac{\partial}{\partial t}(e^s \operatorname{sen} t).$

$= (z_{xx} \cdot x_t + z_{xy} \cdot y_t)(e^s \cos t) + z_x \cdot (-e^s \operatorname{sen} t) +$

$+ (z_{yx} \cdot x_t + z_{yy} \cdot y_t) \cdot (e^s \operatorname{sen} t) + z_y \cdot (e^s \cos t)$

$= [z_{xx} \cdot (-e^s \operatorname{sen} t) + z_{xy} \cdot (e^s \cos t)] \cdot (e^s \cos t) + z_x \cdot (-e^s \operatorname{sen} t) +$

$+ [z_{yx} \cdot (-e^s \operatorname{sen} t) + z_{yy} \cdot (e^s \cos t)] \cdot (e^s \operatorname{sen} t) + z_y \cdot (e^s \cos t)$

Questão 2

a) Calcular $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$.

C_1 : $\underline{x=0}$

$f(0,y) = \frac{0}{y^2} = 0 \Rightarrow f(x,y) \to 0$ ao longo de C_1

C_2 : $\underline{x=y}$

$f(x,x) = \frac{x^2}{2x^2} = \frac{1}{2} \Rightarrow f(x,y) \to \frac{1}{2}$ ao longo de C_2

Logo $\nexists \lim_{(x,y)\to(0,0)} f(x,y)$. Portanto, f não é contínua em $(0,0)$.

b) $\cdot\ (x,y) \neq (0,0)$

$$f_x(x,y) = \frac{y(x^2+y^2) - xy(2x)}{(x^2+y^2)^2} = \frac{y(y^2-x^2)}{(x^2+y^2)^2}$$

$$f_y(x,y) = \frac{x(x^2+y^2) - xy(2y)}{(x^2+y^2)^2} = \frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

$\cdot\ (x,y) = (0,0)$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

$$f_y(0,0) = \lim_{h\to 0} \frac{f(0,0+h)-f(0,0)}{h} = 0$$

$$f_x(x,y) = \begin{cases} \frac{y(y^2-x^2)}{(x^2+y^2)^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}, \quad f_y(x,y) = \begin{cases} \frac{x(x^2-y^2)}{(x^2+y^2)^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

c) Não, pois f não é contínua em $(0,0)$.

Questão 3

(a) $\|\vec{v}\| = \sqrt{(1)^2+(1)^2+(-1)^2} = \sqrt{3}$

$\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{1}{\sqrt{3}}(1, 1, -1)$

$\nabla V(x,y,z) = (10x - 3y + yz, -3x + xz, xy)$

$\nabla V(3,4,5) = (10.3 - 3.4 + 4.5, -3.3 + 3.5, 3.4)$

$= (30 - 12 + 20, -9 + 15, 12)$

$= (38, 6, 12)$

$D_u V(3,4,5) = \nabla V(3,4,5) \cdot \vec{u}$

$= (38, 6, 12) \cdot \dfrac{1}{\sqrt{3}}(1, 1, -1)$

$= \dfrac{1}{\sqrt{3}}(38 + 6 - 12) = \dfrac{32}{\sqrt{3}}$

(b) $\nabla V(3,4,5) = (38, 6, 12)$

(c) $\|\nabla V(3,4,5)\| = \|(38, 6, 12)\| = \sqrt{38^2 + 6^2 + 12^2}$

Questão 4

(a) $\mathrm{Dom} f = \mathbb{R}^2 \setminus \{(0,0)\}$, $\mathrm{Im} f = \mathbb{R}^+_*$

(b) curva de nível $k = \{(x,y) \in \mathrm{Dom} f \mid f(x,y) = k\}$

$= \{(x,y) \in \mathrm{Dom} f \mid \dfrac{1}{x^2+y^2} = k\}$

$= \{(x,y) \in \mathrm{Dom} f \mid x^2+y^2 = \dfrac{1}{k}\}$, $k > 0$

$\boxed{x^2 + y^2 = \dfrac{1}{k}}$

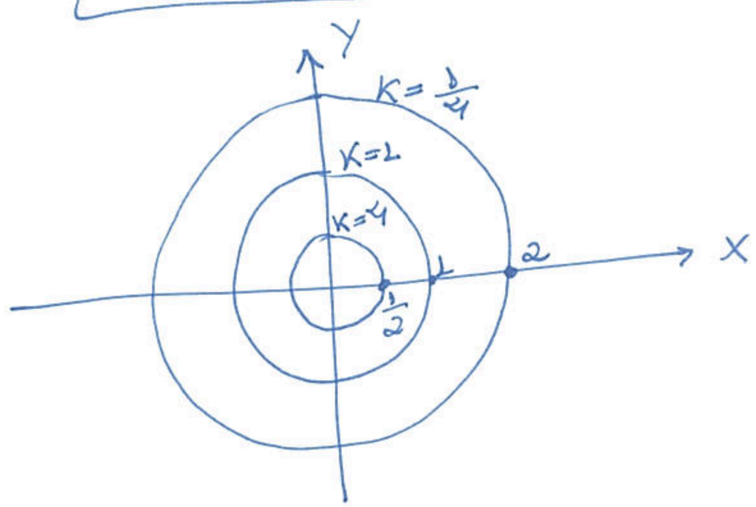
$k > 1 \Rightarrow \dfrac{1}{k} < 1$

$k < 1 \Rightarrow \dfrac{1}{k} > 1$

$k = 1$ $\quad x^2 + y^2 = 1$

$k = 4$ $\quad x^2 + y^2 = \left(\dfrac{1}{2}\right)^2$

$k = \dfrac{1}{4}$ $\quad x^2 + y^2 = (2)^2$



Questão 5

Plano tangente ao gráfico de f em $(a, b, f(a,b))$:

$$\boxed{z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b)}$$

$$f_x(x,y) = \psi\left(\frac{x}{y}\right) + x \cdot \psi'\left(\frac{x}{y}\right) \cdot \frac{1}{y} = \psi\left(\frac{x}{y}\right) + \frac{x}{y}\psi'\left(\frac{x}{y}\right)$$

$$f_y(x,y) = x \cdot \psi'\left(\frac{x}{y}\right) \cdot \left(-\frac{x}{y^2}\right) = -\frac{x^2}{y^2}\psi'\left(\frac{x}{y}\right)$$

$(x,y) = (a,b)$

$$f(a,b) = a\,\psi\left(\frac{a}{b}\right)$$

$$f_x(a,b) = \psi\left(\frac{a}{b}\right) + \frac{a}{b}\psi'\left(\frac{a}{b}\right)$$

$$f_y(a,b) = -\frac{a^2}{b^2}\psi'\left(\frac{a}{b}\right)$$

Equação do plano tangente:

$$z - \left[a \cdot \psi\left(\frac{a}{b}\right)\right] = \left[\psi\left(\frac{a}{b}\right) + \frac{a}{b}\psi'\left(\frac{a}{b}\right)\right](x-a) + \left[-\frac{a^2}{b^2}\psi'\left(\frac{a}{b}\right)\right](y-b).$$

Se $x = y = 0$, o lado ~~esque~~ direito da equação é:

$$\left[\psi\left(\frac{a}{b}\right) + \frac{a}{b}\psi'\left(\frac{a}{b}\right)\right](-a) + \left[-\frac{a^2}{b^2}\psi'\left(\frac{a}{b}\right)\right](-b) =$$

$$= -a\,\psi\left(\frac{a}{b}\right) - \frac{a^2}{b}\psi'\left(\frac{a}{b}\right) + \frac{a^2}{b}\psi'\left(\frac{a}{b}\right) = -a\,\psi\left(\frac{a}{b}\right)$$

O lado esquerdo da equação é $z - \left[a \cdot \psi\left(\frac{a}{b}\right)\right]$.

Se $z = 0$, então o ~~lado~~ lado esquerdo da equação é $-a \cdot \psi\left(\frac{a}{b}\right)$.

Logo $(x, y, z) = (0,0,0)$ satisfaz a equação do plano tangente. Concluímos que ele passa pela origem.